



# Physics Lab 1 Laboratory Textbook

Prepared by the PHYSICS DEPARTMENT for  
DEU Engineering Faculty students.

For Experiment Videos:

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# **Preparation of Experiments for PHYSICS LAB 1**

## **Basic Concepts in Measurement and Uncertainties in Measurement**

DEU Faculty of Science

Physics Department

2021

## 1.1 Measurement

In general, the measurement process can be defined as comparing a quantity with another quantity of the same quality. In other words, it is the work of determining the amount (numerical value) of a physical quantity according to a predetermined standard of the same quality. The predetermined standard is called a **unit**. For example, if the mass of an object is said to be 7 kilograms, it is said that the mass of that object is 7 times the unit defined as 1 kilogram. In other words, measuring a quantity means determining the unit of that quantity and how many times the unit contains.

The tool used in the measurement process is called the **measuring tool**. Measurement tools can be divided into two groups as analog (with display) and digital (digital). Indicative measurement tools are in the form of a ruler divided in units of measurement, and the nearest two lines are called the **scale unit**. In a measuring instrument with a digital display, it can usually be taken as the last digit scale unit (this may not always be the case, it should be determined from the measurement (calibration) values given by the manufacturer.). Scale units for different measuring instruments are shown in Figure 1.1.



Figure 1.1 Measurement tools

We can examine the measurement process in two groups:

**Direct measurement:** Measurements made directly with measuring tools. For example, measuring the length of a pencil with a ruler, measuring the temperature with a thermometer, measuring time with a clock, etc.

**Indirect measurement:** It is a measurement made by calculating a magnitude with the help of directly measurable quantities. For example, measuring the velocity of an object (directly measuring displacement and elapsed time, proportioning displacement to elapsed time), measuring the density of an object (measuring the mass and volume of the object, proportioning the mass to the volume), etc.

Two important concepts to be considered while measuring are accuracy and precision. Accuracy indicates how close a measurement of a physical quantity is to the true value. Precision indicates how close two values obtained from measuring the same magnitude are to each other. Figure 1.2 and Figure 1.3 give the meaning and comparison of the concepts of accuracy and precision.

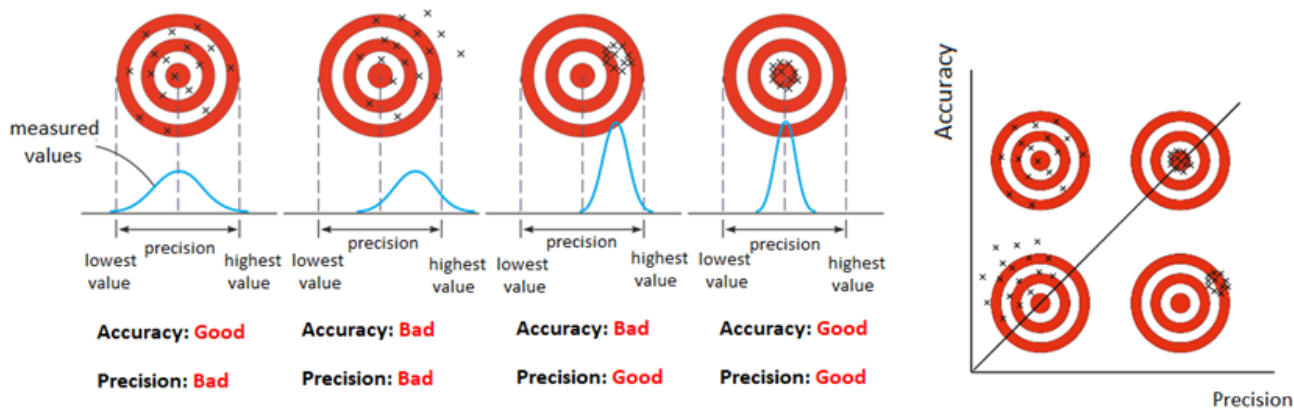


Figure 1.2 The relationship between the concepts of accuracy and precision.

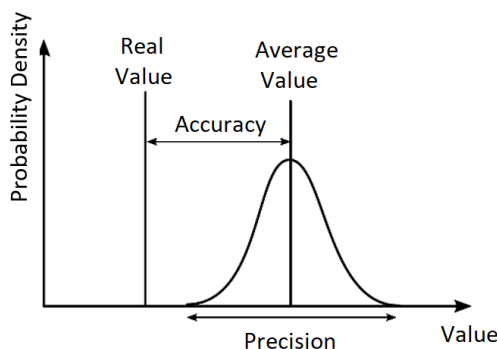


Figure 1.3 The relationship between the concepts of accuracy and precision.

### 1.2 Error (Uncertainty)

The difference between the value of any measured physical quantity and its true value is called error. What is meant by error here is not "wrong" or "defect", but "uncertainty". Depending on the precision of the measuring tool used and the experimental method followed in the measurement, the result of the measurement will be within a certain range. In other words, it will be meaningful if the measurement results are given together with their errors while expressing them. For example, let's measure the quantity of a physical property (length, time, voltage, electric current, ... etc.) of an object as  $x_1$ . The result of  $x_1$  measurement will be within a certain closeness to the true value of  $x$ . If we make a second measurement of  $x_2$ , we will see that this result is slightly different from the result of measuring  $x_1$ . If we make a large number of measurements, we will get a different value for each measurement. Accordingly, it is not possible to determine the true value of  $x$  exactly. For this reason, we can look at the distribution of the measurement results and around which value they are distributed. Including this information, we express the measurement result as follows:

$$\text{Result} = \underbrace{\text{the most probable value}}_{\text{average value}} \pm \frac{\text{Precision}}{\text{error or uncertainty}} \tag{1.1}$$

Here, the most probable value is the value around which the measurements are distributed, which is the value estimated to be closest to the true value.

**Absolute Error:** The difference between the true value  $x$  of a measured physical quantity and the measured value  $x_0$  is called its absolute error.

$$\Delta x = |x - x_0| \rightarrow \text{Result} = x \pm \Delta x \quad 1.2$$

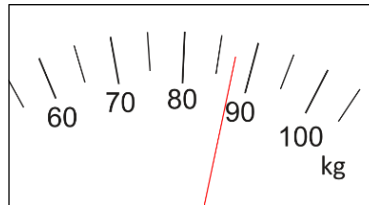
Since the true value is unknown, the absolute error cannot be known either. But its approximate value can be determined by some methods.

**Relative Error:** It is defined as the ratio of the absolute error in the measurement to the true value.

$$\text{Relative Error} = \frac{\Delta x}{x}, \quad \text{Percent Relative Error} = \frac{\Delta x}{x} 100 \quad 1.3$$

### Example 1.1

Express the absolute error, relative error and measurement result in the measurement process given in the figure?



$$m_{avg.} = 87 \text{ kg}, \quad \Delta x = \frac{5}{2} = 2.5 \text{ kg} \rightarrow \text{Result} = 87 \pm 3 \text{ kg}$$

$$\text{Relative Error} = \frac{3}{87}, \quad \% \text{ Error} = 3.4324942 \rightarrow \text{Result} = 87 \text{ kg} \pm 3\%$$

## 1.3 Error Sources

**Systematic Errors:** It is caused by the measuring tools used, the method followed in the experiment and external effects. These errors affect the result in one direction. We can eliminate systematic errors by changing the test method, using more sensitive measuring tools, or making necessary corrections at the end of the experiment. An example of systematic error is given in Figure 1.4. Accuracy can be considered as a systematic error measure that reveals the difference between the measured value and the true value (Figure 1.3).

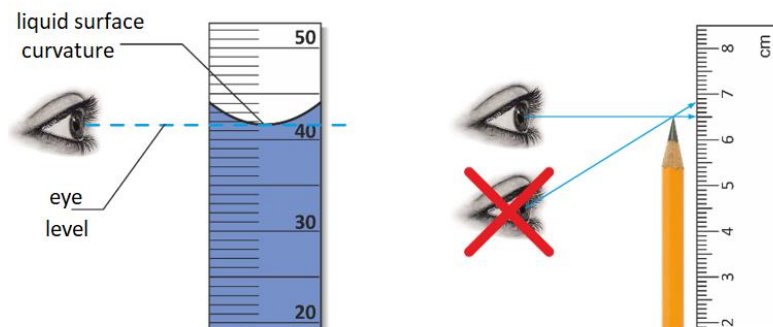


Figure 1.4 Systematic error.

**Statistical (Random) Errors:** Errors resulting from the natural behavior of the measured physical quantity. These errors affect the result both ways. By increasing the number of measurements we can reduce statistical errors and their effect on the accuracy of the measurand can be calculated by statistical analysis. For example, fluctuations in measured physical quantities such as temperature, electrical voltage, and gas pressure cause statistical errors. A comparison of systematic error and statistical error is given in Figure 1.5.



Figure 1.5 Systematic errors and statistical errors

In general, measurement results can be displayed in the following formats.

$$t = (34.5 \pm 0.7) \times 10^{-3} s$$

$$t = 34.510^{-3}s \pm 2\%$$

$$x = 10.3^{+0.7}_{-0.3} cm$$

$$m_e = (0.51099906 \pm 0.000000 15) MeV/c^2$$

$$m_e = 0.51099906 (15) MeV/c^2$$

$$n = 9.109 \times 10^4 mol \pm 0.3 ppm$$

\* ppm: *part per million*

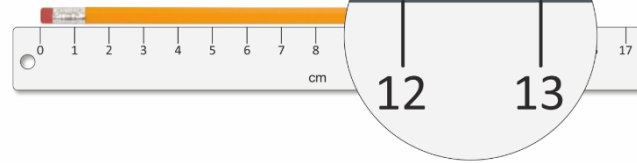
## 1.4 Significant Figures

Numbers written to indicate a measurement result, which are known to be accurate, and the last is guesswork, are significant figures.

### Example 1.2

*Let's say we want to measure the size of a pencil with significant figures. First, let's use a cm scale ruler.*

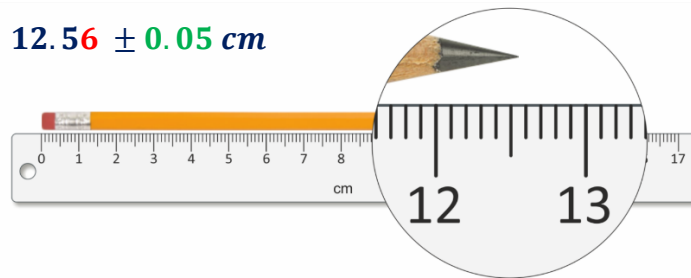
$$L = 12.5 \pm 0.5 \text{ cm}$$



The length of this pen is between 12 cm and 13 cm. 12 is a known number with certainty, it is meaningful. Although the number 5 after the comma is not clear (may be 4 or 7, etc.), it is meaningful because it contains information about the size of the pen. If we were to express the result as 12.57, the number 7 used in the last digit would have no meaning. While the number 5 contains uncertainty, it is not possible to determine the last digit, 7. Therefore, this measurement result can be expressed with 3 significant figures. The maximum value of uncertainty in this process will be half that as the scale unit is cm. That is 0.5 cm.

Now let's say we make the same measurement with a mm scale ruler.

$$L = 12.56 \pm 0.05 \text{ cm}$$



The length of this pen is between 12.5 cm and 12.6 cm. 12.5 is a known number with certainty. The number 6 after the comma is not clear (may be 4 or 7, etc.), but it is meaningful because it contains information about the size of the pen. If we were to express the result as 12,564, the number 4 used in the last digit would have no meaning. While the number 6 contains uncertainty, it is not possible to determine the last digit, 4. Therefore, this measurement result can be expressed with 4 significant figures. The maximum value of uncertainty in this process will be half that as the scale unit is mm. So it is 0.05 cm.

The position of the comma has no significance for significant numbers. The number of significant figures expressed as 0.0565 m 56.5 mm will be 3. The zeros here have no meaning, they just indicate the order of magnitude. While giving a measurement result, not all zeros written may be significant. Figure 1.6 shows which zeros are significant and which are not.

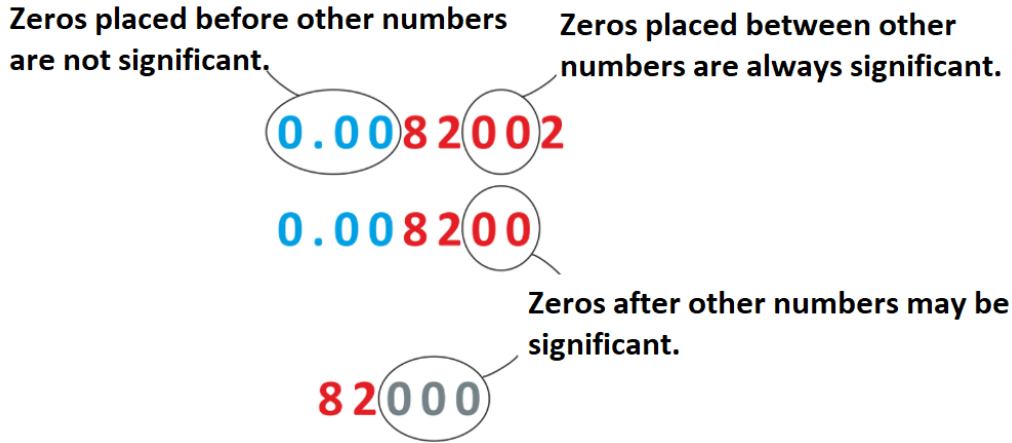


Figure 1.6 Significant figures

So the trailing zeros of a number only matter if they are behind the decimal point. Otherwise, it is difficult to say that they are meaningful. For example, in 8200 measurement results, it is not clear whether the zeros are significant. The number of significant digits in 8200 can be at least two, three or four. To avoid uncertainty, the location of the decimal point should be specified or scientific notation should be used as shown below.

- $8.200 \times 10^3$  four significant numbers
- $8.20 \times 10^3$  three significant numbers
- $8.2 \times 10^3$  two significant numbers

### 1.4.1 Significant Figures and Arithmetic Operations

#### Addition and Subtraction Operations

When adding or subtracting the measured quantities, the precision of the answer is the same as the precision of the least precise term in the sum or difference. All figures up to this precision limit are significant.

Example 1.3	
<i>Let's add up 11.67 cm given at 1/10 cm proximity, 0.25 mm given at 1/100 cm proximity, and 7.4 cm given at cm proximity.</i>	
$  \begin{array}{r}  11.67 \\  0.025 \\  + 7.4 \\  \hline  19.095  \end{array}  $	<p>Among the given numbers, the number with the largest suspect digit is 7.4 cm. Therefore, the result of the procedure should be given in mm. Rounding is done.</p> <p>Result: 19.1 cm (three significant figures)</p>



Measured Values	Maximum	Minimum	The measurement result should be within this range: $19.650 > l > 18.540$
$11.67 \pm 0.05$	11.72	11.62	
$0.025 \pm 0.005$	0.030	0.020	
$7.4 \pm 0.5$	+ 7.9	+ 6.9	
	19.650	18.540	

The most probable value  $x_{avg.} = \frac{19.650+18.540}{2} = 19.095 \text{ cm}$

The uncertainty in addition is  $\Delta x = \frac{19.650-18.540}{2} = 1.11$ . Even the third significant figure is questionable in this example. The result of the measurement is given as  $x = x_{avg.} \pm \Delta x = 19.1 \pm 1.1 \text{ cm}$ .

### Multiplication and Division Operations

As a result of multiplication, the number of significant digits (in some cases, one more) of the factor measured with the least precision is preserved.

#### Example 1.4

If the side lengths of a safe, whose sensitivities are measured with different measuring tools are **25.32 cm**, **30.5 cm** and **10.123 cm**, what is its volume?

$$\text{Volume} = (25.32 \text{ cm}) \times (30.5 \text{ cm}) \times (10.123 \text{ cm}) = 7817.588 \text{ cm}^3$$

The multiplier with the smallest significant number among the multipliers is 30.5 cm. The number of significant digits here is 3. Therefore, the volume should be indicated with 3 (or 4) significant figures.

$$\text{Volume} = 7.82 \times 10^3 \text{ cm}^3 = 7.82 \text{ dm}^3$$

Measured Values	Maximum	Minimum	The measurement result should be within this range: $7965.368 > V > 7670.456$
$25.32 \pm 0.05$	25.37	25.27	
$10.123 \pm 0.005$	10.128	10.118	
$30.5 \pm 0.5$	x 31.0	x 30.0	
	7965.368	7670.456	

The most probable value  $V_{avg.} = \frac{7965.368+7670.456}{2} = 7817.912 \text{ cm}^3$

There are two numbers that are certain at the end of the transaction. (numbers 7...) The numbers that follow are uncertain. But by taking  $7.82 \times 10^3 \text{ cm}^3$ , we get an average value.

The uncertainty in multiplication is  $\Delta V = \frac{7965.368-7670.456}{2} = 147.456 \text{ cm}^3$ . Even the third significant figure is questionable in this example. The result of the measurement is given as  $V = V_{avg.} \pm \Delta V = 7.82 \pm 0.15 \text{ dm}^3$ .

## 1.5 Detection of Errors and Error Propagation

If the value of a physical quantity is to be made as a direct and single measurement, the most probable value is the value that is read and the last one is the value given by the predictive number, and the largest value of the error (uncertainty) in the measurement can be taken as half of the measuring device's scale unit (between the nearest two lines on the measuring device). However, the general preference should be to repeat the measurement. Repeated measurements not only give you a better idea of the true value, but also allow you to characterize the measurement uncertainty. In most cases, the number of repetitions in the laboratory setting is small, usually 5 to 10.

If  $N < 10$ , the most probable value and uncertainty in measurement can be found as follows.

Average Value ( $\bar{x}$ )	It is the average of the measured values. ("most probable value").	$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$
Range (R)	Indicates the distribution range of the measured values. It is the difference between the highest value and the lowest value.	$R = x_{max.} - x_{min.}$
Uncertainty ( $\Delta x$ )	The uncertainty in the measurement process is half the difference between the highest value and the lowest value of the measurement results.	$\Delta x = \frac{R}{2} = \frac{x_{max.} - x_{min.}}{2}$
Uncertainty in Mean Value	The real value of $x$ will be within a range around $x_{avg}$ . As the number of measurements $N$ increases, this range slowly decreases as $(1/\sqrt{N})$ .	$\Delta \bar{x} = \frac{\Delta x}{\sqrt{N}} = \frac{R}{2\sqrt{N}}$
Measurement result	The $x$ measurement result should be shown to include both the mean value and the uncertainty in the mean.	$x = \bar{x} \pm \Delta \bar{x}$

If the effect of random errors on the measurement is dominant, it can be mathematically shown that the measurements are normally distributed by increasing the number of measurements ( $N \rightarrow \infty$ ). The most probable value of the measured quantity is  $x_{avg}$ , and the real value is between  $\bar{x} \pm \sigma_{\bar{x}}$  with 68% probability (Fig. 1.7). If the uncertainty is desired to be given at the 95% confidence interval,  $\Delta x = 1.79\sigma_{\bar{x}}$  should be taken.

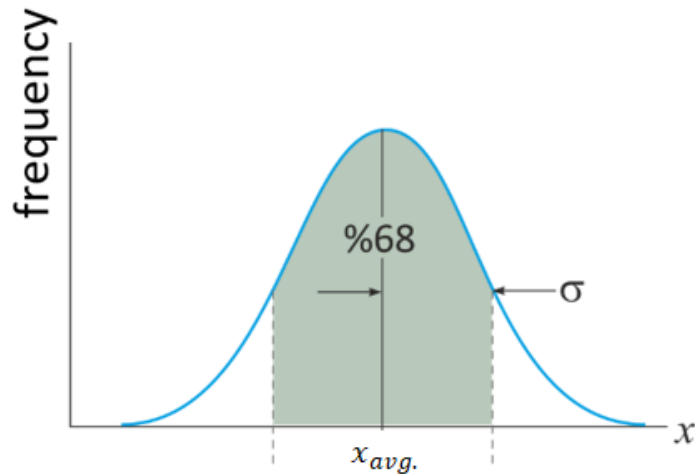


Figure 1.7 Errors that fit the Gaussian distribution.

In case of  $N \gg 10$  (10 -- 102) it would be a more accurate approach to make the calculations as stated below.

Average Value ( $\bar{x}$ )	It is the average of the measured values. ("most probable value").	$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$
Uncertainty ( $\Delta x$ )	Uncertainty in the measurement process; is the standard deviation of the distribution.	$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$
Uncertainty in Mean Value	The real value of $x$ will be within a range around $x_{avg}$ . As the number of measurements $N$ increases, this range slowly decreases as $(1/\sqrt{N})$ .	$\Delta \bar{x} = \frac{\sigma}{\sqrt{N}}$
Measurement result	The $x$ measurement result should be shown to include both the mean value and the uncertainty in the mean.	$x = \bar{x} \pm \Delta \bar{x}$

**Example 1.5**

The period of a simple pendulum is measured 6 times with a 1/10 s precision timer (stopwatch) and the following results are found. Find the mean value of these measurement results and the error (uncertainty) in the mean.

$$t_1 = 3.6 \text{ s}, t_1 = 3.8 \text{ s}, t_1 = 3.6 \text{ s}, t_1 = 3.7 \text{ s}, t_1 = 3.1 \text{ s}, t_1 = 3.3 \text{ s}$$

Average Value:	$\bar{t} = \frac{3.6 + 3.8 + 3.6 + 3.7 + 3.1 + 3.3}{6} = 3.516667 \text{ s}$
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Mean Deviation:

$$\Delta \bar{t} = \frac{|3.6 - 3.5| + |3.8 - 3.5| + |3.6 - 3.5| + |3.7 - 3.5| + |3.1 - 3.5| + |3.3 - 3.5|}{6}$$

$$= 0.216667 \text{ s}$$

$$t = \bar{t} \pm \Delta \bar{t} \rightarrow t = 3.5 \pm 0.2 \text{ s}$$

For simplicity, the mean deviation can be used. However, it does not express well how the mean gets closer to the true value as the number of measurements increases. Therefore, we can say that the calculation made using the following method expresses the measurement result well. Or for  $N > 10$  it would be convenient to take the standard deviation for the uncertainty.

$$R = 3.8 - 3.1 = 0.7 \text{ s}$$

$$\Delta t = R/2 = 0.35 \text{ s}$$

$$\Delta \bar{t} = \frac{\Delta t}{\sqrt{N}} = \frac{0.35}{\sqrt{6}} = 0.142887 \text{ s}$$

$$\Delta \bar{t} = \frac{\sigma}{\sqrt{N}} = \frac{0.26457}{\sqrt{6}} = 0.108012 \text{ s}$$

$$t = 3.5 \pm 0.1 \text{ s}$$

### Example 1.6

The mass of an object was measured 5 times by two different students. Measurements made by two students are given in the table below. Show the measurement results of these students and compare the results.

Measurements	Student 1 (kg)	Student 2 (kg)
$x_1$	72	81
$x_2$	77	80
$x_3$	65	82
$x_4$	85	81
$x_5$	88	79

Student 1:

$$\bar{x} = \frac{72 + 77 + 65 + 85 + 88}{5} = 77.4 \text{ kg}$$

$$R = 88 - 65 = 23 \quad \Delta x = \frac{23}{2} = 11.5$$

$$\Delta \bar{x} = 5.142956 \text{ kg}$$

$$x = 77 \pm 5 \text{ kg}$$

$$\sigma = 8.404761$$

$$\Delta \bar{x} = \frac{8.4048}{\sqrt{5}} = 3.758 \text{ kg}$$

$$x = 77 \pm 4 \text{ kg}$$

Student 2:

$$\bar{x} = \frac{81 + 80 + 82 + 81 + 79}{5} = 80.6 \text{ kg}$$

$$R = 82 - 79 = 3 \quad \Delta x = \frac{3}{2} = 1.5$$

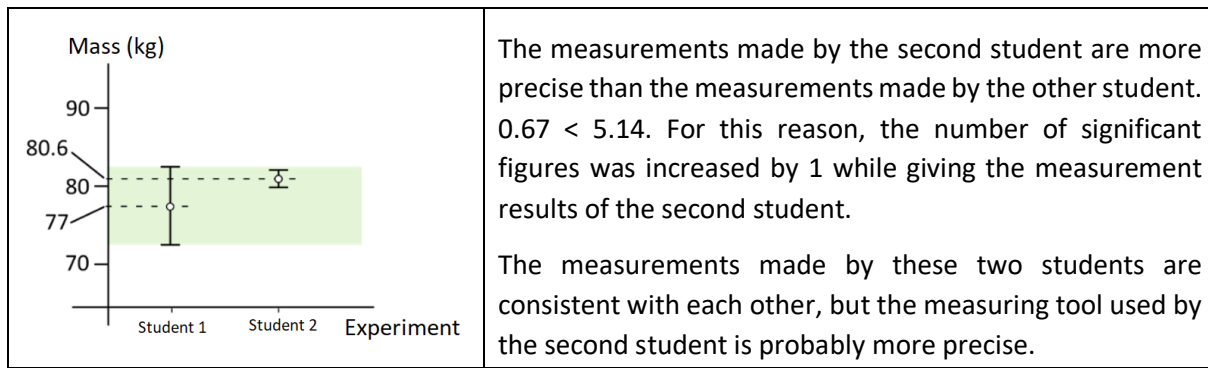
$$\Delta \bar{x} = \frac{1.5}{\sqrt{5}} = 0.67082 \text{ kg}$$

$$x = 80.6 \pm 0.7 \text{ kg}$$

$$\sigma = 1.0198$$

$$\Delta \bar{x} = \frac{1.0198}{\sqrt{5}} = 0.45607 \text{ kg}$$

$$x = 80.6 \pm 0.5 \text{ kg}$$

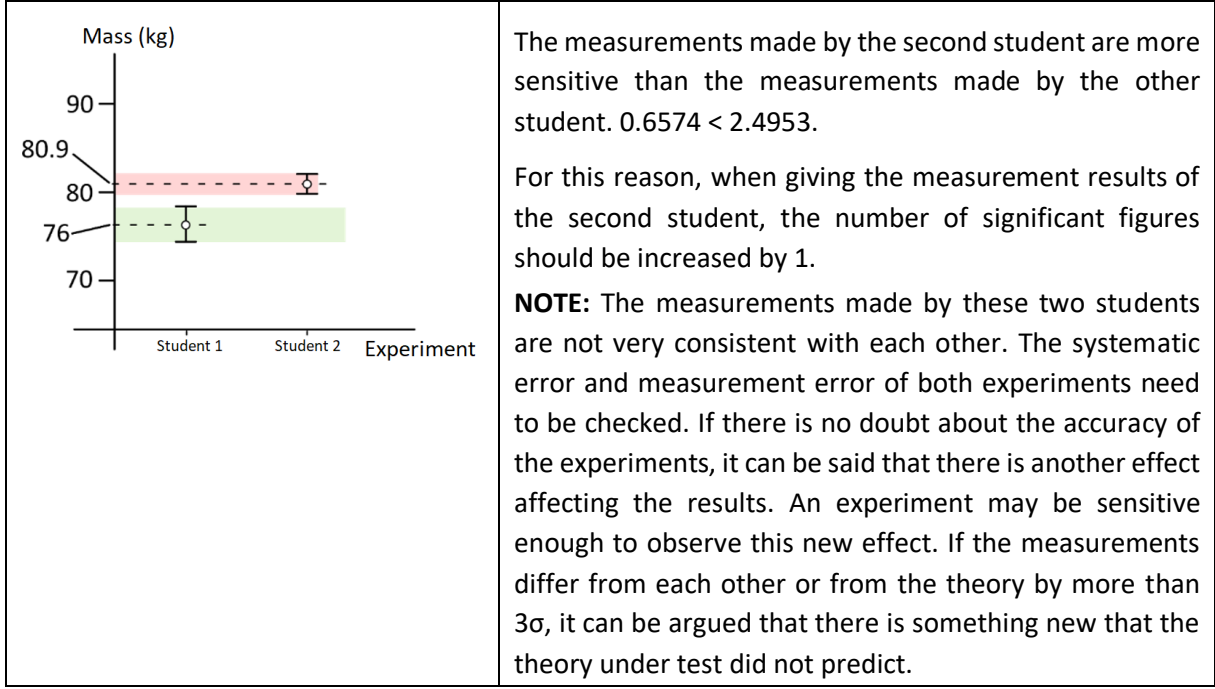


**Example 1.7**

The mass of an object was measured 10 times by two different students. Measurements made by two students are given in the table below. Show the measurement results of these students and compare the results.

Measurements	Student 1 (kg)	Student 2 (kg)
$x_1$	72	81
$x_2$	77	80
$x_3$	65	82
$x_4$	85	81
$x_5$	74	79
$x_6$	88	81
$x_7$	87	83
$x_8$	69	85
$x_9$	76	78
$x_{10}$	71	79

For student 1;	For student 2;
$\bar{x} = 76.4 \text{ kg}$ $\Delta x = \sigma = 7,4859869$ $\Delta \bar{x} = \frac{7,4859869}{\sqrt{10}} = 2,3672769 \text{ kg}$ $x = 77 \pm 2 \text{ kg}$	$\bar{x} = \frac{81 + 80 + 82 + 81 + 79}{5} = 80.9 \text{ kg}$ $\Delta x = \sigma = 1,9723082$ $\Delta \bar{x} = \frac{1,9723082}{\sqrt{10}} = 0,6236986 \text{ kg}$ $x = 80.6 \pm 0.6 \text{ kg}$

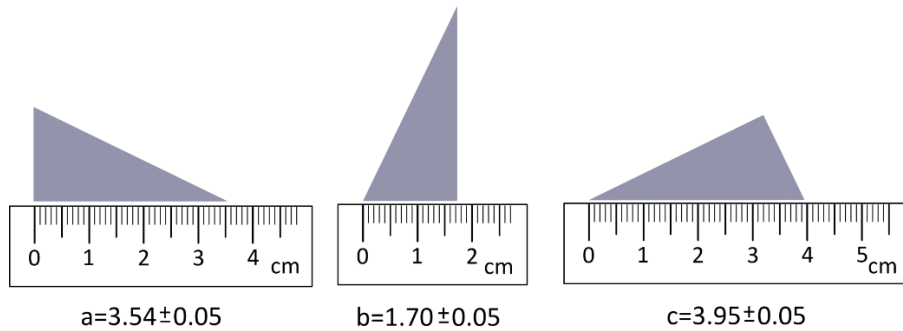


### Error Propagation

We have examined in detail above how to find the most probable value and uncertainty of directly measurable physical quantities. Now, let's imagine that we want to find a physical quantity that cannot be measured directly (density of an object, velocity of an object, perimeter of a triangle). Let the mean of the directly measurable  $x, y, z, \dots$  physical quantities be  $\bar{x}, \bar{y}, \bar{z}, \dots$  and their uncertainty  $\Delta x, \Delta y, \Delta z, \dots$ . The mean value and uncertainty of  $q$  can be found by the following equation.

$$q = f(x, y, z, \dots), \quad \Delta q = \sqrt{\left(\frac{\partial f}{\partial x} \Delta x\right)^2 + \left(\frac{\partial f}{\partial y} \Delta y\right)^2 + \left(\frac{\partial f}{\partial z} \Delta z\right)^2 \dots}, \quad q = \bar{q} \pm \Delta q \quad 1.4$$

As an example, we want to measure the perimeter of a triangular plate (Fig. 1.8).



Şekil 1.8 Üçgenin çevre uzunluğu

$$\bar{C} = a + b + c = (\bar{a} \pm \Delta a) + (\bar{b} \pm \Delta b) + (\bar{c} \pm \Delta c) = \bar{C} \pm \Delta C$$

$$\bar{\zeta} = \bar{a} + \bar{b} + \bar{c} = 3.54 + 1.70 + 3.95 = 9.19 \text{ cm}$$

$\Delta\zeta \neq \Delta a + \Delta b + \Delta c = 1.5 \text{ cm}$  is wrong. It should be calculated from equation 1.4. Accordingly, for addition or subtraction,

$$\Delta\zeta = \sqrt{\Delta a^2 + \Delta b^2 + \Delta c^2} = \sqrt{0.05^2 + 0.05^2 + 0.05^2} = \mathbf{0.086603 \text{ cm}}$$

We get  $\zeta = 9.19 \pm 0.09 \text{ cm}$ .

#### Calculation of errors for some commonly used functions

Operation	Absolute Error (Uncertainty)
$y = a + b$ or $y = a - b$	$\Delta y = \sqrt{\Delta a^2 + \Delta b^2}$
$y = ab$ or $y = a/b$	$\frac{\Delta y}{y} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}$
$y = \lambda a^n$	$\frac{\Delta y}{y} =  n  \frac{\Delta a}{a}$
$y = \lambda \ln \mu a$	$\Delta y = \lambda \frac{\Delta a}{a}$

#### Example 1.8

In a measurement made to find the value of the Pi number, its circumference was measured as 45.2 cm with a measuring tape with 1 cm proximity (cm scale) and its diameter as 14.36 cm with a ruler with 1/10 cm proximity (mm scale). Calculate the number of pi.

$$\bar{\pi} = \frac{\zeta}{R} = \frac{45.2}{14.36} = \mathbf{3.147632}$$

The result can be expressed with 3 or 4 significant figures.

$$\frac{\Delta\pi}{\bar{\pi}} = \sqrt{\left(\frac{\Delta\zeta}{\bar{\zeta}}\right)^2 + \left(\frac{\Delta R}{\bar{R}}\right)^2} \rightarrow \Delta\pi = 3.15 \sqrt{\left(\frac{0.5}{45.2}\right)^2 + \left(\frac{0.05}{14.36}\right)^2} = \mathbf{0.036503}$$

2nd way:

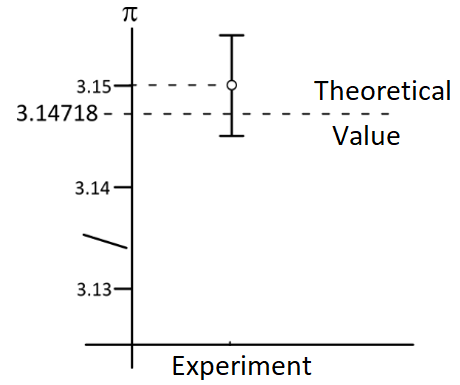
$$\pi_{mak.} = \frac{45.2 + 0.5}{14.36 - 0.05} = 3.193570929$$

$$\pi_{min} = \frac{45.2 - 0.5}{14.36 + 0.05} = 3.102012491$$

$$\pi_{ort} = \frac{\pi_{mak.} + \pi_{min}}{2} = 3.14779171$$

$$\Delta\pi = \frac{\pi_{mak.} - \pi_{min}}{2} = 0.045779$$

$$\pi = \bar{\pi} \pm \Delta\pi = 3.15 \pm 0.04$$



We can say that the result of the experiment is consistent with the value given in the literature. The experiment was carried out successfully.

### Problems

1. Calculate the volume of a sphere measuring  $18.24 \text{ cm}$  with  $1/10 \text{ cm}$  proximity in diameter.
2. Calculate the density of an object with a mass  $m = 24.8 \pm 0.2 \text{ g}$  and a volume  $V = 3.6 \pm 0.4 \text{ cm}^3$ .

### References

1. "An Introduction to Error Analysis", J.R. Taylor, Second edition, University Science Books, 1997.
2. "Fiziksel Ölçmeler ve Değerlendirilmesi", İ., Eşme, Marmara Üniversitesi Yayınları, No:539, 1993.







# Experiment 1

## Basic Measurements and Error Calculation

DEU Faculty of Science  
Physics Department

2021

## 1 Preparation Questions

1. What is measurement? Briefly describe.
2. How to determine the error in direct measurement?
3. How can we measure the speed of an object by giving an example of an indirect measurement process? Subtract the equation expressing the uncertainty in velocity from equation 1.4.

## 2 Aims of the experiment

Learning the basic measurements, which are frequently used in physics, by using sliding caliper, micrometer and electronic scales and making error calculations regarding the measurements taken.

## 3 Theoretical Concepts

### 3.1 Vernier scale

The **vernier scale**, which forms the basis of many tools used to measure length and angle, was invented by Pierre Vernier in 1631 (Figure 2.1). The vernier is an supplementary chart used to accurately determine a fraction of the smallest divisions on the main scale of a measuring tool. In order to understand the vernier measuring tools used, it is necessary to know what the vernier scale is beforehand. Although each segment of the vernier caliper is slightly different from the unit segment of the tool for which it is used to increase its sensitivity, there is a simple correlation between the two segments.

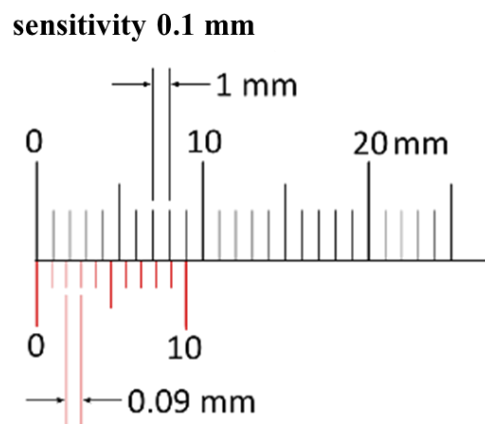


Figure 2.1 Vernier scale

**1) Vernier Caliper:** It is a measuring tool designed according to the vernier measuring system. It consists of a ruler with millimeter divisions and a slider with vernier division that can be slid on it. It is used to measure the dimensions, hole diameter, inner diameter and depth of an object (Figure 2.2).

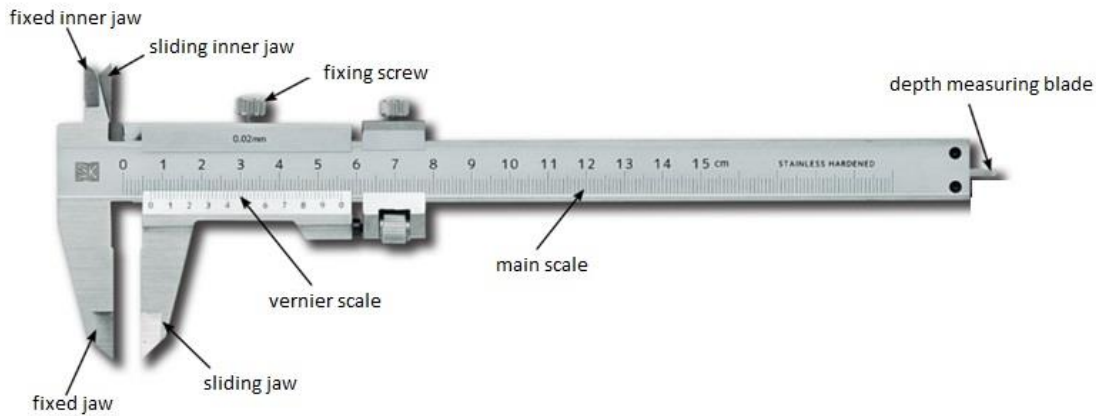
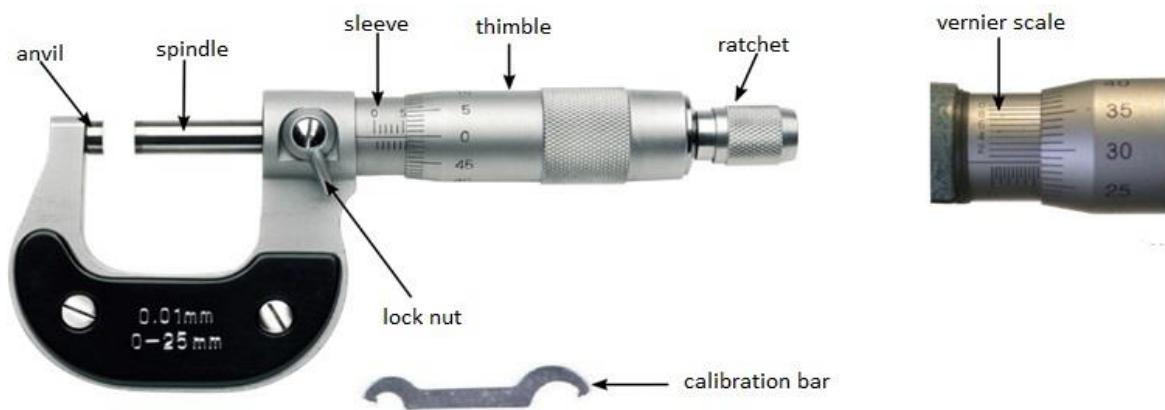


Figure 2.2 Sliding caliper and its parts

With the help of this tool, we can measure lengths in the order of centimeters with an accuracy between 0.1 mm and 0.025 mm, depending on the structure of the tool. On the upper edge of the ruler is the inch scale, which is the English unit of length. The lengths to be measured from outside to outside are placed between the fixed and movable jaws, the lengths to be measured from the inside to the inside are placed in contact with the fixed and movable inner diameter jaws, and the measurement process is performed by making the vernier reading defined above. The sword tip, which moves with the slider, can be used to measure the depth of a hollow object or the height of the water in a container.

**2) Micrometer:** It is a screw measuring instrument that measures 0.01 and 0.001 mm precision (Figure 2.3). As technology develops, the sensitivity of the work done increases. Micrometers are used in order to make more precise and healthier measurements for the works where the measurement accuracy of the calipers is not sufficient. The micrometer is generally produced with a screw pitch of 1 mm or 0.5 mm.



Şekil 2.3 Mikrometer and its parts

The piece to be measured is placed between the fixed and movable jaws of the micrometer and the piece is clamped between the jaws by turning the friction screw (ratchet) until a sound is heard. Since the step of the micrometer shown in Figure 2.4 is 0.5 mm and it is mapped over 50 divisions on the

thimble, the distance between the two lines on the thimble corresponds to  $0.5/50 = 0.01 \text{ mm}$ . This value forms the scale unit of the micrometer. The measurement result is added to the value of the last line visible on the socket (Figure 2.4b, 7.5 mm) by multiplying the value of the horizontal line on the socket where the horizontal line cuts the scale on the thimble (21.7) by the scale unit (0.01 mm) ( $7.5 + 21.7 \times 0.01 = 7.717 \text{ mm}$ ). The maximum value of the absolute error in measurement can be taken as half of the scale unit ( $0.01/2 = 0.005 \text{ mm}$ ).

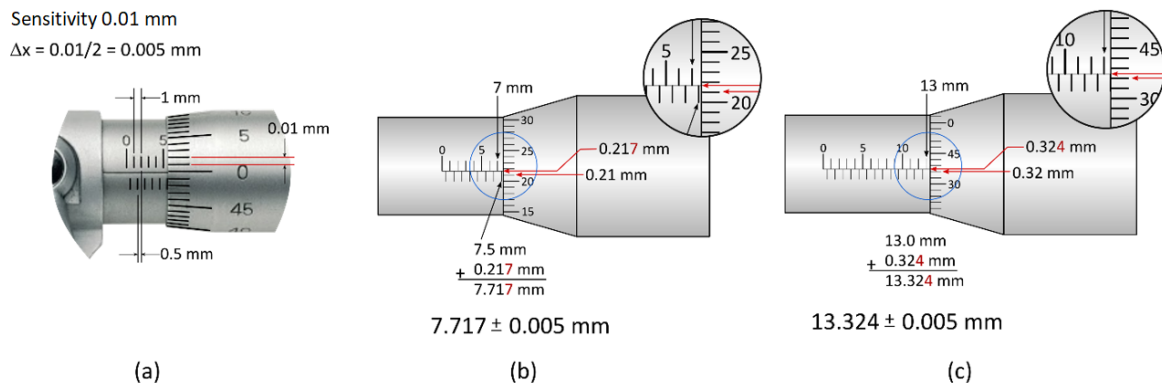


Figure 2.4 Mikrometer<sup>1</sup> scales, uncertainty and measurement procedures

**Important note about using micrometer:** If the parts of the micrometer are not well understood, the measurements may be inaccurate or the micrometer may be damaged. For example, when clamping the piece to be measured between the jaws, it is necessary to use the friction screw (ratchet), not the thimble. Since the ratchet spring applies a compression force of approximately 2-2.5 N, the pressure will always remain the same no matter who uses the micrometer. As a result, errors due to printing (deformation) will be prevented. In addition, squeezing from the thimble can damage the mechanism of this precision measuring tool. This point should be considered in order not to reduce the working life of the micrometer and to prevent measurement errors.

In the vernier scale seen in Figure 2.4a, the 10 divisions of the vernier scale correspond to the 9 divisions of the main scale. Therefore, each vernier division is  $1/10$  smaller than a main division. The first vernier section is  $1/10$  units to the left of the first section of the main scale, the second vernier section is  $2/10$  units to the left from the second main section, etc. For this reason, the 10th vernier division coincided with the 9th main division.

Figure 2.5 shows a measurement made. The result is an exact value seen on the 12 mm major scale. On the vernier scale, the closest overlapping line is seen as the 5th and 6th lines. In this case, there is uncertainty. Therefore, 5 and 6 can be treated as a predicted number. **It would be necessary to take half of the sensitivity for the measure of uncertainty in the measurement, but here we can say that the uncertainty would be 0.05 mm larger. It would not be correct to write the result as  $x = 12.6? \pm 0.05 \text{ mm}$ .** For this reason, it would be more accurate to take the maximum value of the uncertainty in measurement as 0.1 mm. In this case, the measurement result should be given as  **$12.6 \pm 0.1 \text{ mm}$** . Different vernier scales and measurement examples are given in Figure 2.6.

<sup>1</sup> <https://www.makinaegitimi.com/mikrometreler/>

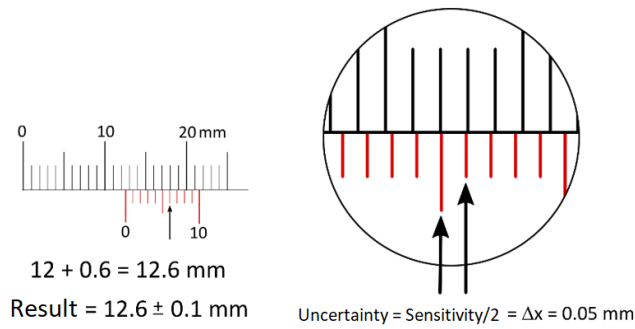


Figure 2.5 Vernier scale and measurement process.

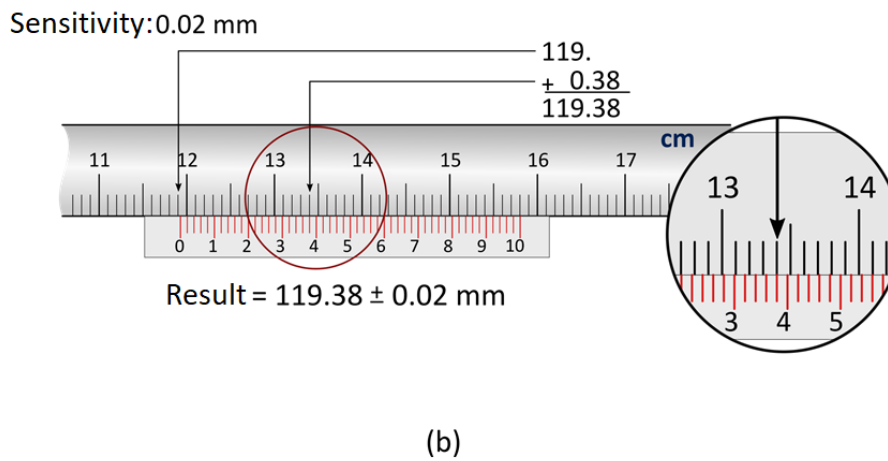
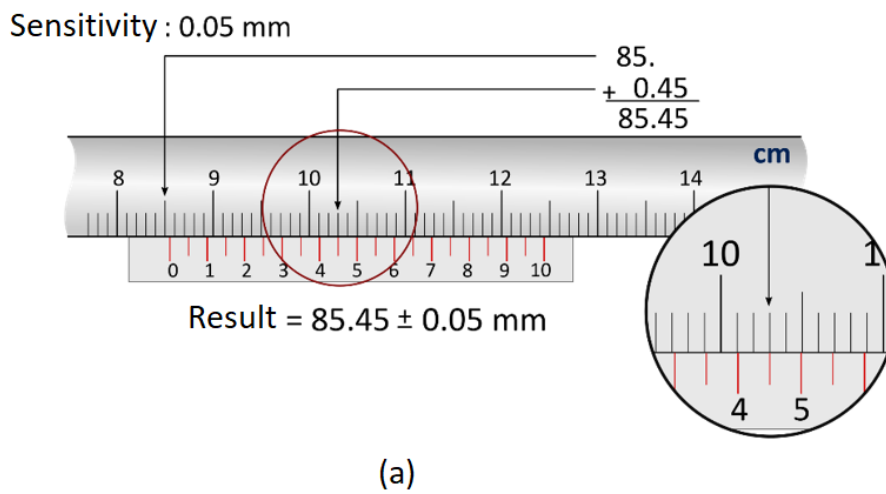


Figure 2.6 Vernier scale and measurement process.

## 4 Materials

Sliding caliper, micrometer with 0.5 mm pitch, electronic balance, various objects of different geometries, Beaker.

## 5 Experimental Procedure

### 5.1 An object with uniform geometry

a) Fill in Table 2.1 below by measuring the lengths of different edges of a piece of wood with a uniform geometry 10 times using a suitable length scale (micrometer or caliper).

**Table 2.1** Measurement results (write in mm)

$i$	$x_i$	$x_i - \bar{x}$	$(\Delta x_i)^2$	$y_i$	$y_i - \bar{y}$	$(\Delta y_i)^2$	$z_i$	$z_i - \bar{z}$	$(\Delta z_i)^2$
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									

$q$ :	$x$	$y$	$z$
$\bar{q} = \frac{1}{N} \sum_{i=1}^N q_i$			
$\sigma = \sqrt{\frac{\sum_{i=1}^N (\Delta q_i)^2}{N-1}}$			
$\Delta \bar{q} = \frac{\sigma}{\sqrt{N}}$			
$\bar{q} \pm \Delta \bar{q}$	$\pm$	$\pm$	$\pm$

**b.** Determine the volume of this object, the uncertainty in the volume ( $q = xyz$  in Equation 1.4) (absolute error) and the relative error.

$$\bar{V} = \quad \text{cm}^3, \quad \Delta V = \quad \text{cm}^3, \quad \frac{\Delta V}{V} =$$

$$V = \quad \pm \quad \text{cm}^3$$

**c.** Weigh the mass of the object on the balance. (The sensitivity of the scales we use in our laboratory is 0.1 grams.)

$$m = \quad \pm \quad g$$

**d.** Determine the density of this object, the uncertainty in its density ( $q = m/V$  in Equation 1.4) (absolute error), and the relative error.

$$\bar{\rho} = \quad \frac{g}{\text{cm}^3}, \quad \Delta \rho = \quad \frac{g}{\text{cm}^3}, \quad \frac{\Delta \rho}{\rho} =$$

$$\rho = \quad \pm \quad \frac{g}{\text{cm}^3}$$

## 5.2 An object with non-uniform geometry

For the volume of an object with non-uniform geometry, immerse the beaker full of water so that the object is completely submerged. The rising amount of water in the beaker will give the volume of the object. By measuring its mass on a balance, we can find its density by proportioning the mass to the volume. The uncertainty in measuring the volume of the object will be half the volume difference between the two closest lines on the chart on the beaker. Fill in the blanks below by making these calculations.

$$V_B = \quad \pm \quad \text{cm}^3, \quad V_D = \quad \pm \quad \text{cm}^3$$

$$V_{Taş} = \quad \text{cm}^3, \quad \Delta V_{Taş} = \quad \text{cm}^3, \quad \frac{\Delta V_{Taş}}{\bar{V}_{Taş}} =$$

$$V_{Taş} = \quad \pm \quad \text{cm}^3$$

$$m = \quad \pm \quad g$$

$$\bar{\rho} = \quad \frac{g}{\text{cm}^3}, \quad \Delta \rho = \quad \frac{g}{\text{cm}^3}, \quad \frac{\Delta \rho}{\rho} =$$

$$\rho = \quad \pm \quad \frac{g}{\text{cm}^3}$$



## 6 Questions

1. How can you reduce the standard deviation of the measurements you take?
2. Which measuring instrument should have the smallest absolute error when comparing caliper and micrometer? Why?
3. What could be the sources of error in your measurements?
4. The thimble of one of the two micrometers with a pitch of 1 mm is mapped over 50 divisions and the other over 100 divisions. Which of these micrometers should be used to make more precise measurements? Why?



# Experiment 2

## Vectors (Force Table)

DEU Faculty of Science

Physics Department

2020

26

### 3.1 Preparation Questions

1. What is a vector? Compare vector quantities and scalar quantities with a few examples.
2. How to find the composition of two or more vectors? Briefly mention the methods.
3. What is a unit vector?
4. State the differences between the force and mass by considering the differences between vectors and scalars.
5. What are the conditions for a system to be in equilibrium? Explain briefly.

### 3.2 Aims of the experiment

To determine the force which balances two other forces experimentally by the help of the force table.

To verify the resulting forces by adding components of the two forces and also by using graphical method.

### 3.3 Theoretical Concepts

#### 3.3.1 Scalar and vector quantities

The quantities in physics can be divided into two groups as "scalar" and "vector" quantities.

A scalar quantity is completely specified by a single value with an appropriate unit and has no direction. For example, mass, volume, time, temperature, pressure, potential energy, kinetic energy, and etc.

A vector quantity is completely specified by a number with an appropriate unit (the magnitude of the vector) and a direction. Displacement, velocity, acceleration, force, momentum, electric field, magnetic field are examples of vector quantities. Figure 3.1 shows the components of a vector that need to be known. The straight line  $xy$  on which the vector is located is called as the direction of the vector. The magnitude (intensity) of the vector is equal to the length of the arrow ( $|AB|$ ) and denoted by  $|\vec{a}|$  or  $a$ . The arrow shows the direction of the vector.

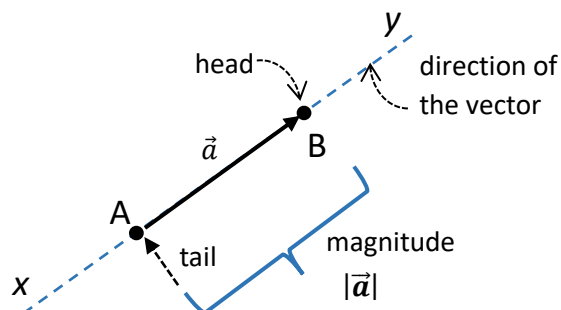


Figure 3.1 The definition of a vector

## Vector Operations

### Vector addition:

The vector sum of all of the forces acting on a body is a single force called as resultant force. Vector addition can be done in three ways: Experimental method, graphical method and component method.

#### a) **Experimental Method:**

Two forces are applied on the force table by hanging masses over pulleys positioned at certain angles. Then the angle and mass hung over a third pulley are adjusted until it balances the other two forces. This third force is called as equilibrant (restoring) force ( $\vec{F}_E$ ) since it is the force which establishes equilibrium. The equilibrant force is not the same as the resultant force ( $\vec{F}_R$ ). The resultant is the vector sum of the two forces. While the magnitude of the equilibrant force is equal to the magnitude of the resultant force, its direction is in the opposite direction of the resultant force. Because it balances the resultant force (see Figure 3.2). So, the equilibrant force  $\vec{F}_E$  is the negative of the resultant force  $\vec{F}_R$ .

$$-\vec{F}_E = \vec{F}_R = \vec{F}_A + \vec{F}_B$$

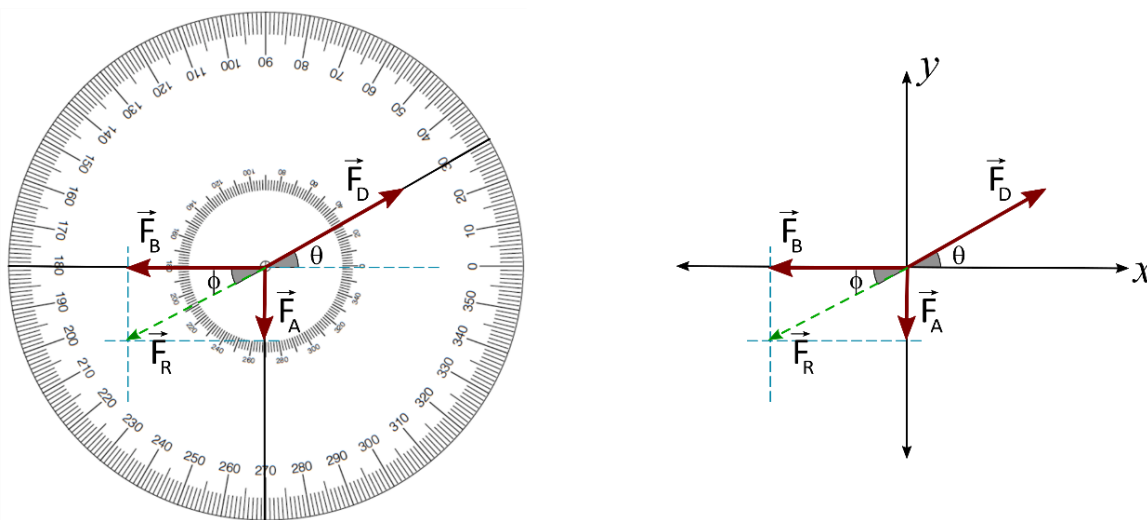


Figure 3.2 The equilibrant vector and the resultant vector

#### b) **Graphical Method:**

In this method, two forces are added together by drawing them to scale using a ruler and protractor. They can be added together by drawing a line from head to tail (Figure 3.3). Graph paper can be used for drawing. The starting point of the second force ( $\vec{F}_B$ ) is drawn by joining the endpoint ( $\vec{F}_A$ ) of the first force. Then the magnitude of the resultant force ( $|\vec{F}_R|$ ) can be measured on the diagram. The magnitude of the force

can be found based on the chosen scale. Also,  $\theta$  angle can be measured with a protractor.

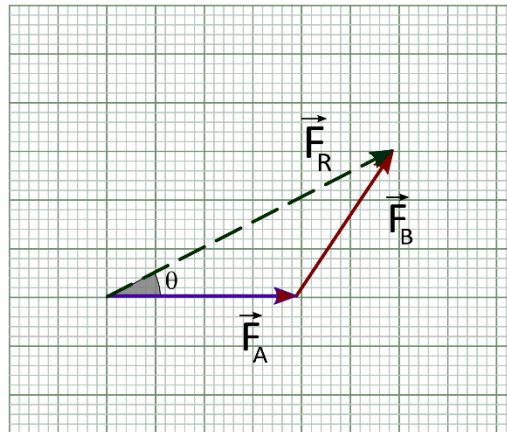


Figure 3.3 Graphical method

**c) Unit Vectors and Component Method:**

Vector quantities often are expressed in terms of unit vectors. A unit vector is a dimensionless vector quantity having a magnitude of 1. Its purpose is to describe a certain direction in space.

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}, \quad |\hat{a}| = 1$$

In an  $xy$ -coordinate system we can define a unit vector “ $\hat{i}$ ” that points in the direction of the positive  $x$ -axis and a unit vector “ $\hat{j}$ ” that points in the direction of the positive  $y$ -axis. The unit vectors  $\hat{i}$  and  $\hat{j}$  form a set of mutually perpendicular vectors in a right-handed coordinate system as shown in Figure 3.4(a). The magnitude of each unit vector equals 1; that is,  $|\hat{i}| = |\hat{j}| = 1$ . We can write a vector  $\vec{A}$  lying in the  $xy$  plane in terms of its component (Figure 3.4(b)).

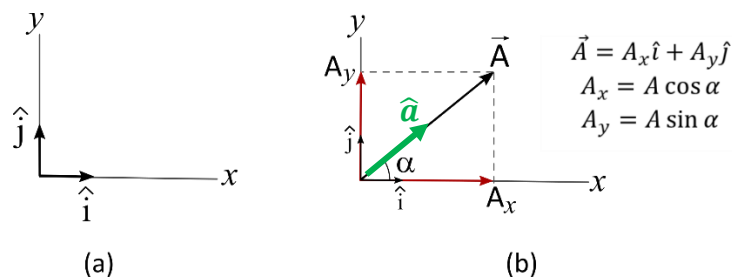


Figure 3.4 a) Unit vectors in  $xy$ -plane b) The expression of vector  $\vec{A}$  in terms of its components

**Using Component Method to Calculate Resultant Vector:**

In this method, the resultant force  $\vec{F}_R$  can be found by adding the  $x$  and  $y$  components, which are the projections of the forces on a selected set of axes, separately. To add vectors (Figure 3.5), the two forces must first be decomposed into their  $x$ - and  $y$ -components:

$$\vec{F}_A = F_{Ax}\hat{i} \quad \vec{F}_B = -F_{Bx}\hat{i} + F_{By}\hat{j}$$

- Here,  $F_{Ax}$  is the x-component of the vector  $\vec{F}_A$  and  $F_{Bx}$  is the x-component of the vector  $\vec{F}_B$ .  $F_{By}$  is the y-component of the vector  $\vec{F}_B$ .  $\hat{i}$  is the unit vector in the x-direction and  $\hat{j}$  is the unit vector in the y-direction (Figure 3.5). Here,  $F_{Ax}$ ,  $F_{Bx}$  and  $F_{By}$  are all positive numbers.

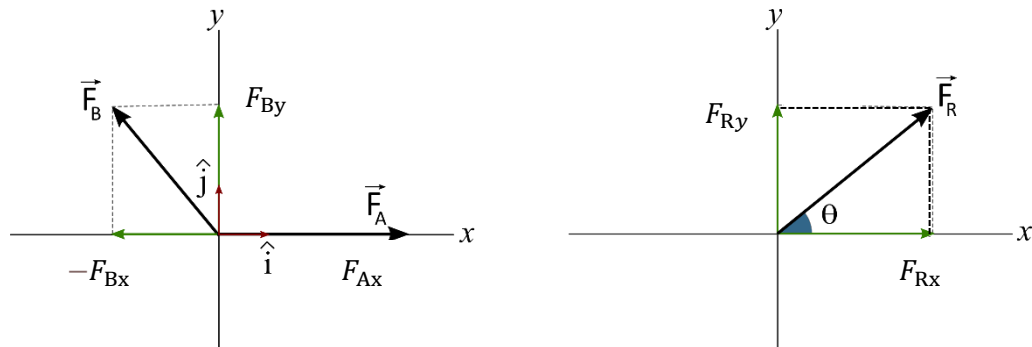


Figure 3.5 Finding resultant vector by component method

By adding the components of the two forces  $\vec{F}_A$  and  $\vec{F}_B$ , the components of the resultant force  $\vec{F}_R$  can be obtained as follows:

$$\vec{F}_R = (F_{Ax} - F_{Bx})\hat{i} + F_{By}\hat{j} = F_{Rx}\hat{i} + F_{Ry}\hat{j}$$

To calculate the magnitude of the resultant force, the components of the resultant ( $R_x$  and  $R_y$ ) must be combined using the Pythagorean Theorem since the components are at right angles to each other.

$$|\vec{F}_R| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

The angle between the x-axis can be calculated using trigonometry:

$$\tan(\theta) = \frac{F_{Ry}}{F_{Rx}}$$

### Other Methods to find equilibrant force, resultant force

**Lami's Theorem** : Lami's Theorem states, "when three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces". Referring to the below diagram in Figure 3.6, consider three forces  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$  acting on a particle or rigid body making angles  $\alpha$ ,  $\beta$  and  $\gamma$  with each other.

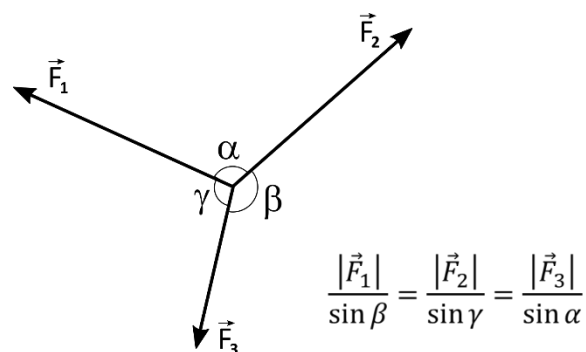


Figure 3.6 Lami's theorem (the law of sines).

**The Law of Cosines:** The law of cosines is used for calculating the third side of a triangle when the other two sides and their enclosed angle are known, and in calculating the angles of a triangle if all three sides are known. The Law of Cosines is a generalization of the Pythagorean Theorem.

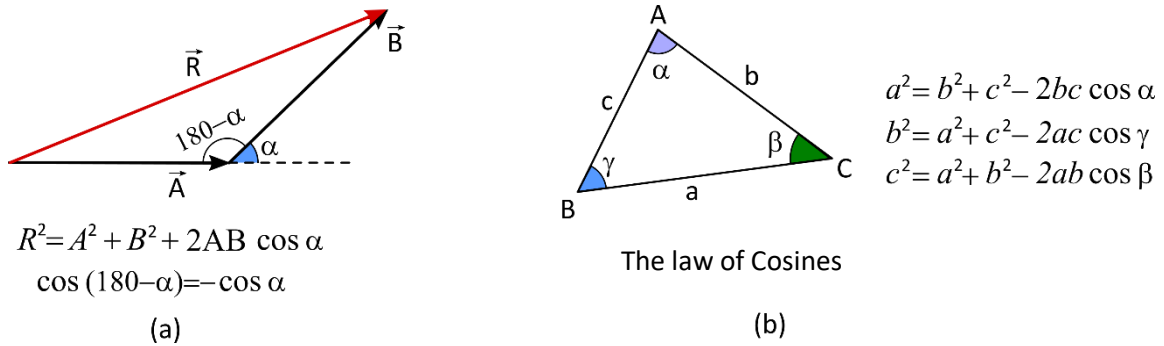


Figure 3.7 The law of Cosines.

### 3.4 Materials

Force table, pulleys with clamps (3), mass hangers (3), plastic ring, mass set and string

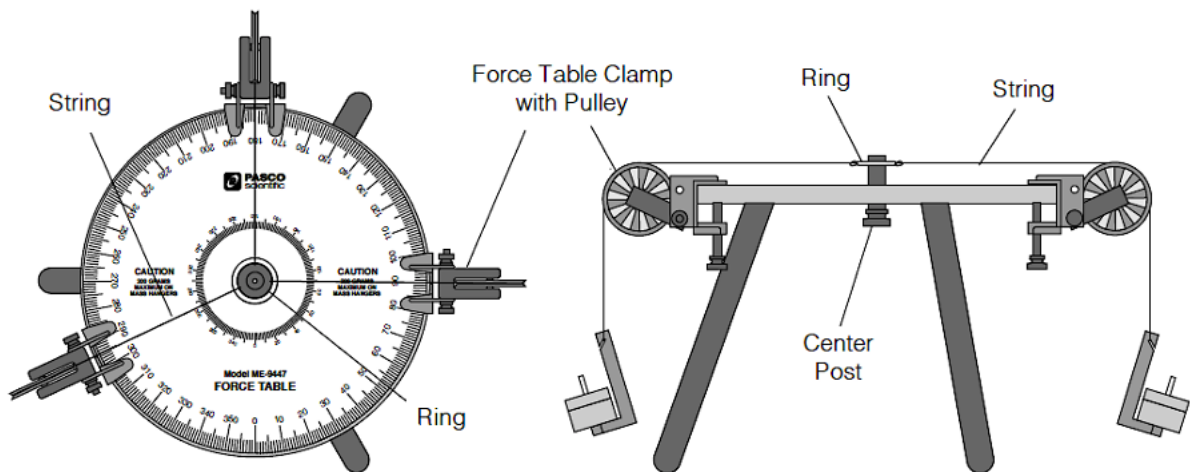


Figure 3.8 Experimental setup of force table



### 3.5 Experimental Procedure

- 1- Assemble the force table as shown in Figure 3.8. Place three pulleys on the force table. Two of them are for the forces that will be added and the remaining one is for the force that balances the sum of the other two forces. Then, screw the center post up until it stops so that it sticks up above the table. Place the ring over the post and tie one string (at least 30 cm long) to the ring for each pulley. The strings must be long enough to reach over the pulleys. Place each string over a pulley and tie a mass hanger to it (A string can be attached to the mass hanger by wrapping the string several times (4 or 5) around the notch at the top of each mass hanger). Make sure that the ring in the center does not touch the surface of the force table, and make sure that the ropes you pass over the pulley are tied at 90 degrees to the ring. Please don't forget to check the angle between the string and the plastic ring as the pulleys change their position on the force table.
- 2- Hang random masses over two of the pulleys and clamp the pulleys at arbitrary angles.
- 3- By trial and error, find the angle for the third pulley clamp and the mass that must be suspended over the pulley so that its weight will balance the forces exerted on the strings by the other two masses.
- 4- Please use the following ring method to determine if the system is in balance.  
 The ring should be centered over the post when the system is in equilibrium (or when you look at the force table from above, the projection of the ring should overlap with the black ring drawn in the middle of the force table). Screw the center post down so that it is in alignment with the top surface of the force table and no longer able to hold the ring in position. Pull the ring slightly to one side and let it go. Check to see that the ring returns to the center. If not, adjust the mass and/or angle of the pulley until the ring always returns to the center when pulled slightly to one side.
- 5- Record the angles of the pulleys on the force table in Table 3.1
- 6- Weigh the masses that hang over each pulley on an electronic scale. Record the mass values you read in Table 3.1 and calculate the corresponding forces.
- 7- Draw the vectors corresponding to these forces in Figure 3.9
- 8- To determine theoretically what mass should be suspended from the third pulley, and at what angle, calculate the magnitude and direction of the equilibrant force by the component method. Please specify the components of the vectors you have drawn in Figure 3.9 on the same figure. For this, decompose the two forces you applied into x and y components and record the relevant component values in Table 3.2. Using these results, find the components of the resultant force and calculate the magnitude of the resultant force.
- 9- The third force, which is called as the equilibrant force ( $\vec{F}_E$ ), is the negative of the resultant force.
 
$$-\vec{F}_E = \vec{F}_R = \vec{F}_1 + \vec{F}_2$$
 Record the equilibrant force you obtained experimentally, the theoretically calculated resultant force and the angle values of these forces with the x (or y) axis in Table 3.3.
- 10- Compare the results obtained by the two methods. Are the results different? Please explain the reason.
- 11- Calculate the absolute error and relative error in the experiment.

12- Repeat the experiment for different masses and different angle values.

Table 3.1 Results of the measurement

Pulley 1			Pulley 2			Pulley 3		
$m_1(\text{kg})$	$ \vec{F}_1  \text{ (N)}$	$\alpha(^{\circ})$	$m_2(\text{kg})$	$ \vec{F}_2  \text{ (N)}$	$\beta(^{\circ})$	$m_3(\text{kg})$	$ \vec{F}_D  \text{ (N)}$	$\theta (^{\circ})$

Table 3.2 Vector components

Results (x-components)	1 <sup>st</sup> Force $F_{1x} \text{ (N)}$	2 <sup>nd</sup> Force $F_{2x} \text{ (N)}$	3 <sup>rd</sup> Force (equilibrant force) $F_{Ex} \text{ (N)}$	Resultant force (Theoretical Result) $F_{Rx} \text{ (N)}$	Absolute error	Relative error
1st measurement						
2nd measurement						
Results (y-components)	1 <sup>st</sup> Force $F_{1y} \text{ (N)}$	2 <sup>nd</sup> Force $F_{2y} \text{ (N)}$	3 <sup>rd</sup> Force (equilibrant force) $F_{Ey} \text{ (N)}$	Resultant force (Theoretical Result) $F_{Ry} \text{ (N)}$	Absolute error	Relative error
1st measurement						

<b>2nd measurement</b>						
------------------------	--	--	--	--	--	--

Note: The error (uncertainty) in the calculation of components can be calculated using the following equations.

$$F_x = F \cos \alpha \rightarrow \Delta F_x = \sqrt{\left(\frac{\partial F_x}{\partial F} \Delta F\right)^2 + \left(\frac{\partial F_x}{\partial \alpha} \Delta \alpha\right)^2} = \bar{F}_x \sqrt{\left(\frac{\Delta F}{\bar{F}}\right)^2 + \left(\frac{\sin \alpha}{\cos \alpha} \Delta \alpha\right)^2}$$

$$F_y = F \sin \alpha \rightarrow \Delta F_y = \sqrt{\left(\frac{\partial F_y}{\partial F} \Delta F\right)^2 + \left(\frac{\partial F_y}{\partial \alpha} \Delta \alpha\right)^2} = \bar{F}_y \sqrt{\left(\frac{\Delta F}{\bar{F}}\right)^2 + \left(\frac{\cos \alpha}{\sin \alpha} \Delta \alpha\right)^2}$$

Table 3.3 Comparison of the resultant force from calculation and the equilibrant force from experiment

	1 <sup>st</sup> Measurement		2 <sup>nd</sup> Measurement	
	$ \vec{F}_R $ (N) (Theoretical)	$ \vec{F}_E $ (N) (Experimental)	$ \vec{F}_R $ (N) (Theoretical)	$ \vec{F}_E $ (N) (Experimental)
Magnitude of the force				
Angle with the x or y axis $\theta$ (°)				
Absolute error in force				
Relative error in force				
Absolute error in angle				
Relative error in angle				

### 3.6 Questions

1. How do the theoretical values for the direction and magnitude of the resultant force compare with the actual direction and magnitude of the *equilibrant force*?

2. Which method gives more reliable results among the results you found with the two methods? Please explain the reason.
3. What are the sources of error in the experiment?
4. Could the magnitude of a particle's displacement be greater than the length of the displacement taken?
5. Can you find two vectors of different lengths but with a vector sum of zero?
6. If the vector sums of three vectors are to be zero, what length conditioning is required for these three vectors? Please explain.

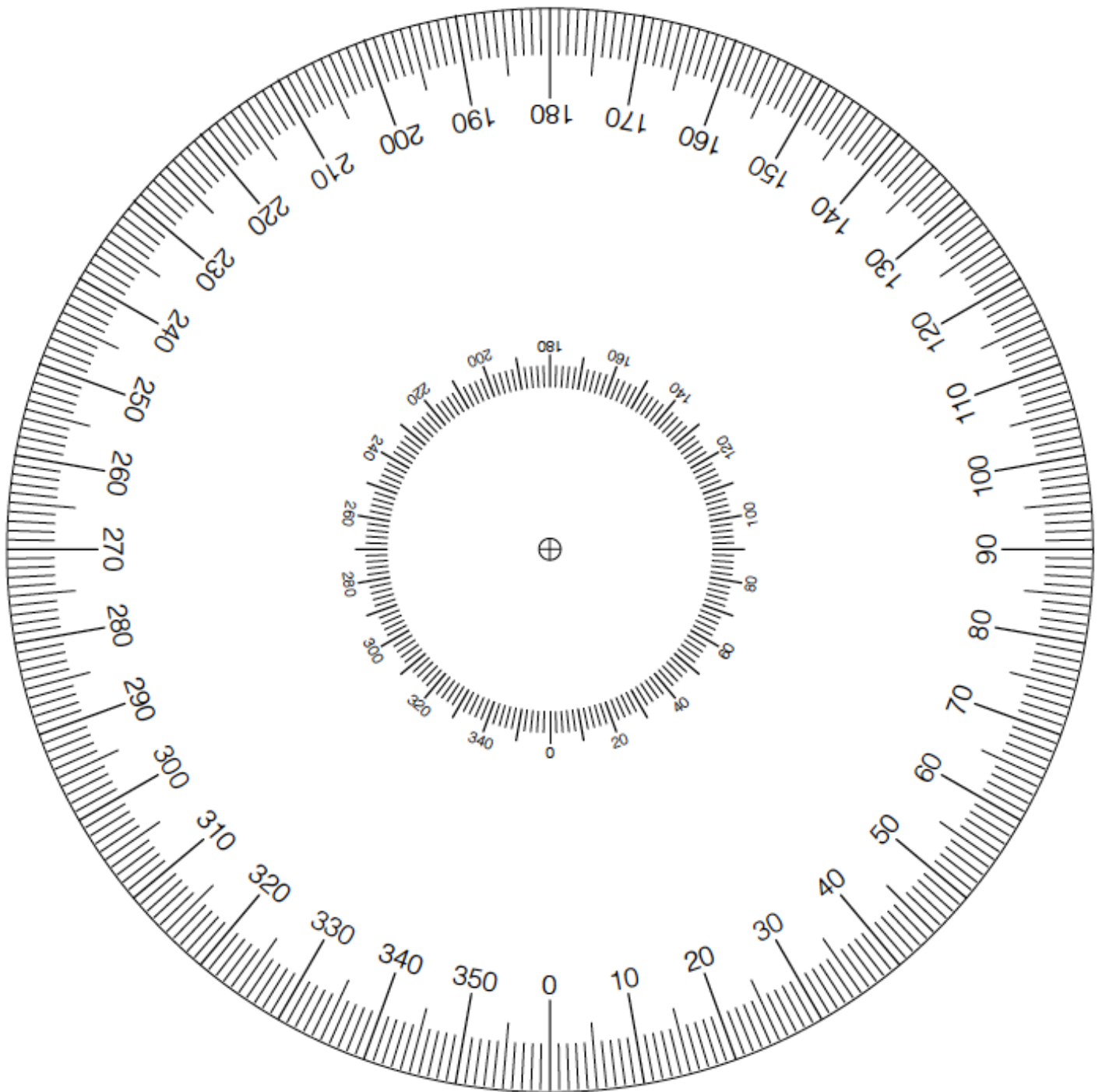


Figure 3.9 Protractor Template





## **3rd Experiment**

# **One-Dimensional Motion**

**DEU Faculty of Science  
Physics Department**

2020

## 4.1 Preparation Questions

1. Average velocity and instantaneous velocity are often different quantities. Is there a motion situation where these two quantities are equal?
2. Consider the movement of marble in your hand, (i) when it is dropped from the rest from a certain height, (ii) when it is thrown downward, (iii) when it is thrown upward. What can be said about the acceleration of the ball for these three motions?

## 4.2 Aims of the Experiment

- To examine free fall motion, which is an example of constant acceleration motion in one dimension,
- To determine the magnitude of the gravitational acceleration  $g$  that causes this motion.

## 4.3 Theoretical Concepts

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was Galilei who discovered this fact for the first time. There is a legend that he demonstrated the law of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. From careful measurements of length and time intervals, it has been observed that the displacement of an object moving from a stationary state is proportional to the square of the time passed through the movement of the object. This observation is consistent with one of the kinematic equations obtained for constant acceleration motion. Galileo's achievements in mechanical science have an important role in the development of Newton's laws of motion. Suppose a coin and a crumpled-up piece of paper drop from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time (In a real experiment, air resistance cannot be neglected). In the idealized case, in which air resistance is absent, such motion is referred to as **free fall**. If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, such a demonstration was conducted on the Moon by astronaut David Scott. He simultaneously released a hammer and a feather, and they fell to the lunar surface at the same time. This demonstration confirmed Galileo's postulation and would have surely pleased Galileo.



The constant acceleration of a freely falling body is called the acceleration due to gravity, and its magnitude is denoted with the letter **g**. The approximate value of **g** at the Earth's surface is  $9.81 \text{ m/s}^2$ , and decreases with increasing altitude. The **g** vector is directed downward, towards the center of the Earth.

A freely falling body is not just a falling body starting from a rest state. A freely falling body is a body falling only under the influence of gravity, regardless of its initial motion.

Equations of motion of a freely falling body, depending on the first motion;

$$v = v_0 - gt$$

$$y - y_0 = \frac{1}{2}(v + v_0)t$$

$$y - y_0 = v_0t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Position-time, velocity-time and acceleration-time graphs for a freely falling body are shown in Figure 4.1.

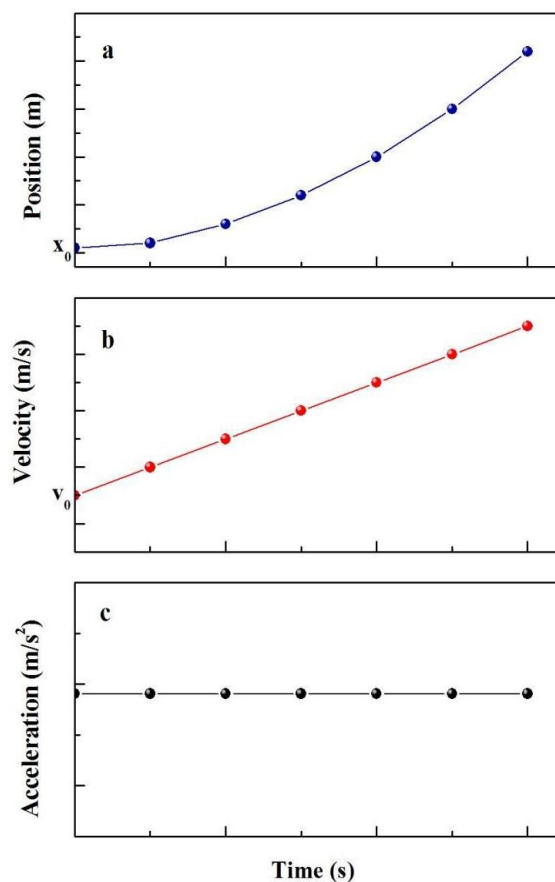


Figure 4.1 (a) Position vs. time, (b) velocity vs. time and (c) acceleration vs. time graphs.

#### 4.4 Tools to be used in the experiment

- Photogate
- Picket fence
- Computer and Datastudio program

**Photogate:** It consists of a source emitting Infra-red radiation on one side and a receiver that can detect this radiation on the other. Thus, when the light is cut off, the sensor will send a signal to the computer and the computer will determine the time until the next interruption.

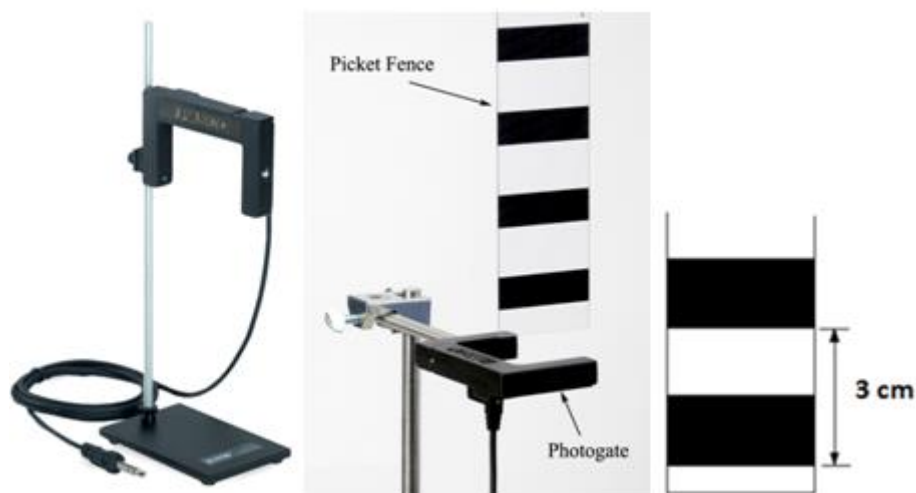


Figure 4.2 Photogate and picket fence

**Picket Fence:** It is a clear plastic strip with uniformly spaced black bands. The fence used consists of 1 cm wide black bands at 2 cm intervals. Each black band blocks the photogate beam as the fence falls through the photogate. Thus, the computer gives the time elapsed between the two banded parts of the fence (3 cm part).

#### 4.5 Experimental Procedure

**Datastudio:** Launching “Datastudio” program providing data collection:

- Double-click the DataStudio icon on your desktop to launch the DataStudio software; Select channel 1 or 2 the photogate is connected to. “Choose sensor or instrument” section will appear, select the section “Photogate and Picket fence”.
- Click the “Position” button in the “Measurements” section. Change the “bandgap” as 0.030 in the “constants” section.
- You can open the Position-Time table by clicking the “Position” button at the “Table” section.

- Hold the Picket Fence at one end vertically on the photogate, so that when it is released, it will fall straight without rotating. Make sure the bottom edge of the Picket Fence is just above the Photogate beam when you let go. Click the “Start” button to begin collecting data. Start recording data, then drop the Picket Fence through the Photogate. Stop recording data.
- You should have 10 entries in the data table (These are the times that each of the 10 bands on the Picket Fence broke the Photogate beam. You should take another run if you have less entry than ten).
- Note down the position-time data to the Table.
- Calculate the average speed of the Picket Fence during the time between the first and second block. Repeat the calculations for further blocks. Enter your calculated average speeds into the table. You will end up with only 9 average speeds in your table.

$$v_1 = \frac{x_2 - x_1}{t_2 - t_1}, \quad v_2 = \frac{x_3 - x_2}{t_3 - t_2} \dots$$

- Calculate the average acceleration of the Picket Fence using the speed values (It will be 8 average acceleration).

$$a_1 = \frac{v_2 - v_1}{t_2 - t_1}, \quad a_2 = \frac{v_3 - v_2}{t_3 - t_2} \dots$$

- Calculate the average gravitational acceleration “g”.
- Calculate your relative error in the experiment.
- Create a position vs. time, speed vs. time and acceleration vs. time graphs using the data-table.

Table 4.1 Experimental Results

	Time (s)	Position (m)	Speed (m/s)	Acceleration (m/s <sup>2</sup> )
1				
2				
3				
4				
5				
6				
7				
8				

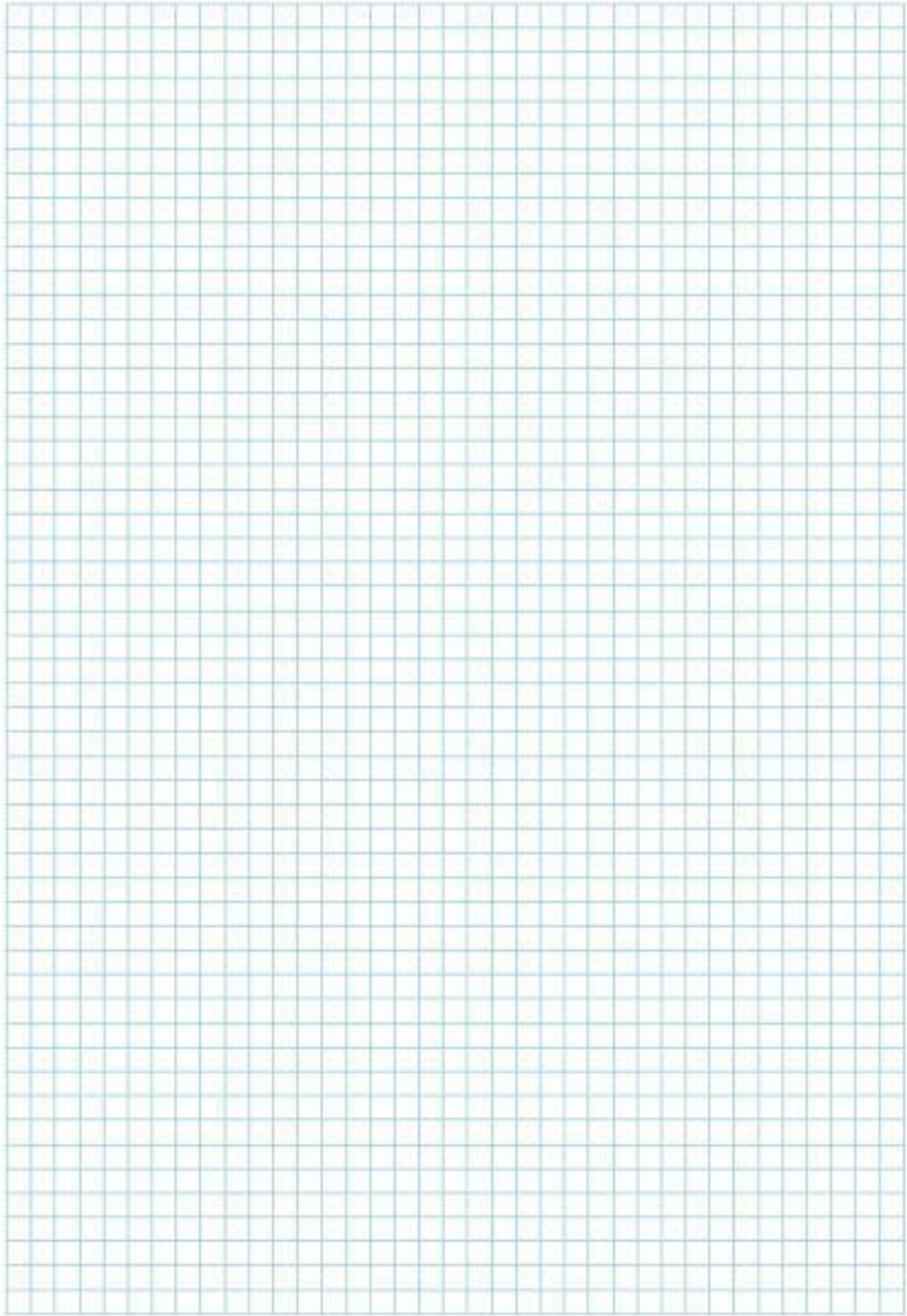
9				
10				

$$g = \underline{g} \pm \frac{\sigma}{\sqrt{N}} \quad (N = 8)$$

$$g = (\dots \pm \dots) \frac{m}{s^2} \quad \% \text{ Relative error} = \dots$$

#### 4.6 Questions

1. Are the speeds you found average or instantaneous? Please explain.
2. If the same experiment was done at poles and equator, how would the gravitational acceleration value change? Why?





## Experiment 4

### Motion in Two Dimensions (Projectile Motion)



DEU Faculty of Science

Physics Department

2020

## 5.1 Preparation Questions

1. Define two-dimensional motion (projectile motion). What are the characteristics of projectile motion? Give examples of projectile motion.
2. Mention about the importance of the angle in projectile motion briefly.
3. In which point on its trajectory of the projectile motion the speed of the object is the lowest?

## 5.2 Aims of the Experiment

To investigate the motion of an object thrown at an angle with the ground (projectile motion) in a two-dimensional plane.

To determine the initial velocity of the object making the projectile motion, the maximum height it can reach, the elapsed time to get the maximum height and the speed of hitting the ground

## 5.3 Theoretical Concepts

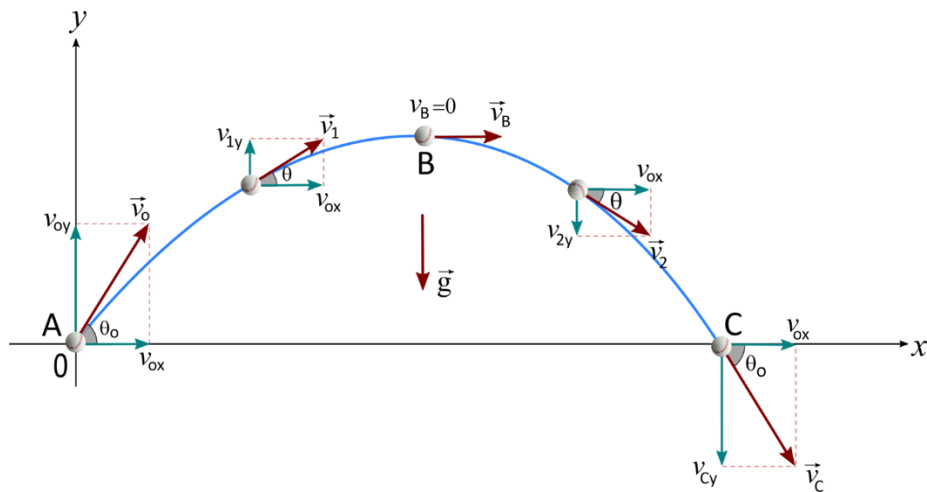
The motion of an object is named as linear, curvilinear or circular, depending on the path it follows. A projectile is any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its trajectory. To analyze this common type of motion, we'll start with an idealized model, representing the projectile as a particle with an acceleration (due to gravity) that is constant in both magnitude and direction. We'll ignore the effects of air resistance.

Projectile motion is always confined to a vertical plane determined by the direction of the initial velocity (Fig. 5.1). This is because the acceleration due to gravity is purely vertical; gravity can't accelerate the projectile sideways. Thus projectile motion is *two-dimensional*. We will call the plane of motion the *xy*-coordinate plane, with the *x*-axis horizontal and the *y*-axis vertically upward.

Galileo was the first person to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately.

- the particle is moving with constant velocity in the horizontal direction (acceleration  $a_x = 0$ )
- the particle is moving with a constant acceleration in the vertical direction ( $a_y = -g$ , where  $g = 9.81 \text{ m/s}^2$ )





**Figure 5.1** Projectile motion of a small ball projected horizontally with initial velocity  $\vec{v}_0$  at angle  $\theta_0$  to the horizontal. The blue line represents the path of the object.

To analyze the two-dimensional motion of a ball, we assume that the motion begins at time at the origin of an x-y coordinate system ( $x_0 = y_0 = 0$ ) and the initial velocity of the ball is  $\vec{v}_0$ . The velocity vector is in the direction of motion at each point, and thus is tangent to the path (Figure 5.1).

When the ball is first thrown, the y-component of the initial (shooting) velocity ( $v_{0y}$ ) has the maximum value. Because of the downward acceleration of gravity, the upward component of velocity ( $v_y$ ) gradually decreases with time until the object reaches the highest point on its path, at which point  $v_{2y} = v_{By} = 0$ . Subsequently the object moves downward (Figure 5.1) and  $v_y$  increases in the downward direction, as shown (that is, becoming more and more negative). During this motion, the x-component of the velocity remains constant ( $v_{Ax} = v_{Bx} = v_{Cx} = v_{0x}$ ), because there isn't any force acted on the ball through x-direction. In other words, acceleration  $a_x = 0$  in the horizontal direction.

As shown in Figure 5.1, in order to obtain the velocity vector at any moment (at any point on the trajectory), the horizontal and vertical velocity vectors at that point must be added by means of geometric methods. We assumed that at time  $t = 0$  the ball is at the point ( $x_0 = y_0 = 0$ ) and that at this time its velocity components have the initial values  $v_{0x}$  and  $v_{0y}$ . The initial velocity of the ball, and the components of the velocity in the horizontal and vertical directions are expressed as follows

$$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} \quad (5.1)$$

$$v_{0x} = v_0 \cos(\theta_0) \quad v_{0y} = v_0 \sin(\theta_0) \quad (5.2)$$

Thus the x-component of acceleration is  $a_x = 0$ , the x-component of the velocity throughout its motion is constant

$$v_x = v_{0x} = v_0 \cos(\theta_0)$$

The displacement of the ball on the x-axis at any time t ( $x_0 = 0$ ):

$$x = x_0 + v_{0x}t = v_{0x}t \quad (5.3)$$

Since the y-component of acceleration is constant and non-zero,  $v_y$  changes in equal amounts over equal time intervals. In projectile motion, the intention of the vertical component of the acceleration is equal to the gravitational acceleration and is in the negative y direction ( $a_y = -g$ ) with respect to the selected reference.

$$v_y = v_{0y} + a_y t = v_{0y} - gt \quad (5.4)$$

The displacement of the ball on the y-axis at any time t ( $y_0 = 0$ ):

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = v_{0y}t - \frac{1}{2}gt^2 \quad (5.5)$$

At the top of the trajectory (point B), the y component of the velocity is zero ( $v_{By} = 0$ ). The time from the moment the shot is made to the moment the ball reaches the maximum vertical height (rising time) is obtained as follows.

$$v_{0y} = gt_{rising} \Rightarrow t_{rising} = \frac{v_{0y}}{g} = \frac{v_0 \sin(\theta_0)}{g} \quad (5.6)$$

At the end point of the trajectory (point C) the y position is zero ( $y = 0$ ).

$$y_C = v_{0y}t - \frac{1}{2}gt^2 = 0 \quad (5.7)$$

If the above equation (Eqn. 5.7) is solved for t, the time of flight ( $t_{flight}$ ) can be obtained as follows:

$$t(v_{0y} - \frac{1}{2}gt) = 0$$

$$t_{flight} = \frac{2v_{0y}}{g} = \frac{2v_0 \sin(\theta_0)}{g} \quad (5.8)$$

So, the landing time of the ball from point B to point C is defined as

$$t_{landing} = t_{flight} - t_{rising} = 2 \frac{v_{0y}}{g} - \frac{v_{0y}}{g} = \frac{v_{0y}}{g} = \frac{v_0 \sin(\theta_0)}{g} = t_{rising}$$

As a result, the time for the ball to climb to the B point and to fall the level where it was thrown is the same.

The maximum height that ball can rise can be found via following equations:

$$y_{max} = v_{0y}t_{rising} - \frac{1}{2}gt_{rising}^2$$

$$y_{max} = \frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2(\theta_0)}{2g} \quad (5.9)$$

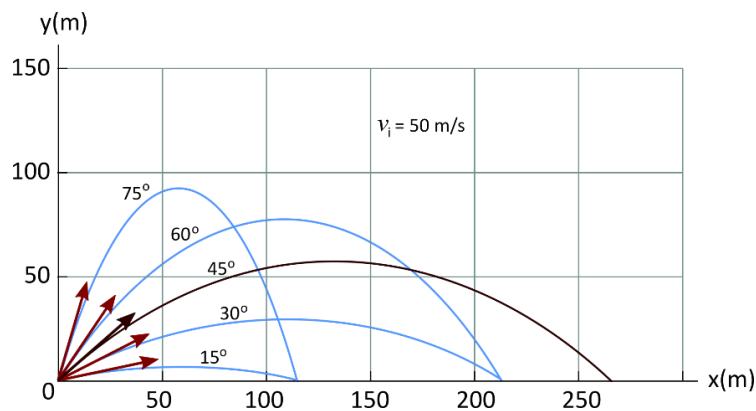
Since the horizontal velocity and total flight time of the object are known, the distance (the path it takes along AC) taken by the ball on the horizontal axis, in other words, the "horizontal range" can be calculated:

$$x_{range} = R = v_{0x} t_{flight} = v_0 \cos(\theta_0) \frac{2v_0 \sin(\theta_0)}{g}$$

$$x_{range} = R = \frac{v_0^2 \sin(2\theta_0)}{g} \quad (5.10)$$

\*The half-angle identity for sine:  $\sin(2\theta_0) = 2 \sin(\theta_0) \cos(\theta_0)$

Figure 5.2 shows the trajectories of an object thrown at a certain initial speed and at different angles. The range of the object thrown at any angle is the same for angles that complement each other to  $90^\circ$  degrees.



**Figure 5.2** Object thrown from the origin at a certain initial velocity at different angles.

The range ( $x_{range}$ ) reaches its maximum when the object is thrown at an angle of  $45^\circ$  degrees with the horizontal:

$$R_{maksimum} = \frac{v_0^2}{g} \quad (5.11)$$

The velocity of the ball at point C can be calculated for  $t = t_{flight}$  moment:

$$v_{Cy} = v_{0y} - g t_{flight} = v_{0y} - g \frac{2v_{0y}}{g} = -v_{0y}$$

$$v_{Cx} = v_{0x}$$

The speed of the ball hitting the ground when the ball falls to the same height ( $y = 0$ ) it was thrown and the angle when it hits the ground:

$$v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = \sqrt{v_{0x}^2 + (-v_{0y})^2} = v_A \quad (5.12)$$

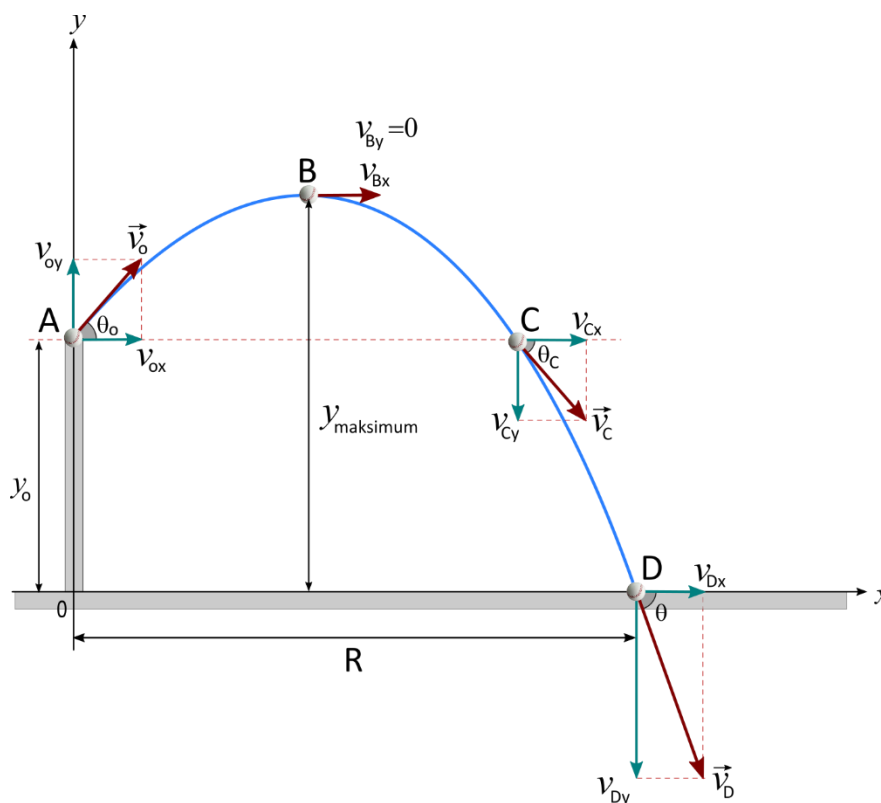
$$\theta_C = \tan^{-1} \left( \frac{v_{Cy}}{v_{Cx}} \right) = \tan^{-1} \left( \frac{-v_{0y}}{v_{0x}} \right) = -\theta_0 \quad (5.13)$$

Therefore, we can say that the velocity of the ball at point C has the same magnitude as the initial velocity, but the y component is reversed and makes the same angle  $\theta_0$  **with** the x – axis in clockwise direction (negative direction).

The equations will change when the ball starts to thrown projectile motion from a certain height ( $y_0 \neq 0$ ) as shown in Figure 5.3, not from the origin. Because the ball will fall to a different point (D) from the level it was thrown (A).

The time it takes for the ball to climb to the B point and the time to fall from B to D is different. Only the elapsed time for AB and BC paths is equal.

$$t_{rising} = t_{AB} = \frac{v_{0y}}{g} = \frac{v_0 \sin(\theta_0)}{g}$$



**Figure 5.3** The path followed by an object that starts an projectile motion from a certain height.

The changes of the x and y components of the position and velocity vectors of this ball will be as follows:

X-components

$$v_x(t) = v_0 \cos(\theta_0) \quad (5.14)$$

$$x(t) = v_0 \cos(\theta_0) t \quad (5.15)$$

Y-components

$$v_y(t) = v_0 \sin(\theta_0) - gt \quad (5.16)$$

$$y(t) = y_0 + v_0 \sin(\theta_0) t - \frac{1}{2}gt^2 \quad (5.17)$$

The maximum height expression that the ball can reach can be obtained from the Equation 5.17 ( $y(t_{rising}) = y_{max}$ ):

$$y_{max} = y_0 + v_0 \sin(\theta_0) t_{rising} - \frac{1}{2}g(t_{rising})^2$$

$$y_{max} = y_0 + \frac{v_0^2 \sin^2(\theta_0)}{2g} \quad (5.18)$$

The time between AD can be obtained from the Equation 5.17 ( $y(t_{flight}) = 0$ ).

$$0 = y_0 + v_0 \sin(\theta_0) t_{flight} - \frac{1}{2}gt_{flight}^2$$

$$t_{flight} = \frac{v_0 \sin(\theta_0)}{g} + \frac{\sqrt{v_0^2 \sin^2(\theta_0) + 2y_0g}}{g} \quad (5.19)$$

The distance from the D point to the origin ( $R = x_{range} = v_{0x} t_{flight}$ ) can be obtained from the Equation 5.15.

$$R = v_0 \cos(\theta_0) t_{flight} \quad (5.20)$$

## 5.4 Tools to be used in the experiment

- Photogate  
(It consists of a source emitting Infra-red radiation on one side and a receiver that can detect this radiation on the other. Thus, when the light is cut off, the sensor will send a signal to the computer and the computer will determine the time until the next interruption.)
- Time-of-Flight-Accessory  
(It is used with a photogate to capture time of flight data. Timing starts when object passes through photogate and stops when object impacts this landing pad.)
- Projectile Launcher  
(It is a mechanism that enables shooting at different inclination angle values in the range of  $0^\circ - 90^\circ$  (measured from horizontal). The angle is easily adjusted using thumbscrews and the built-in protractor and plumb-bob give an accurate way to measure the angle of inclination. And the Projectile Launchers has three range settings. The Short Range, Medium Range and Long Range. The Long-Range Launcher has a stronger spring than Medium Range and Short Range. During this experiment, the "Short Range" will be used.)
- Science Workshop Interface 500

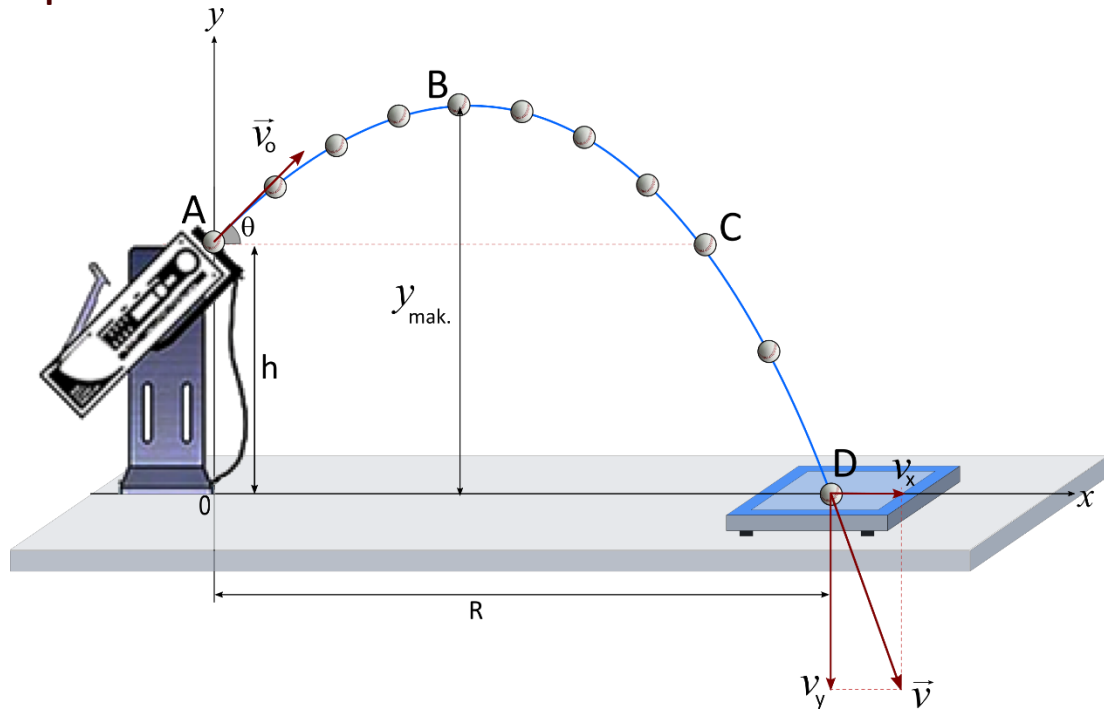
(Interface for connecting time-of-flight-accessory and photogate with computer)

- Plastic balls, Ruler, Plastic Rod
- Computer and Data Studio software



**Figure 5.4** Projectile Motion Set: Projectile launcher, plastic rod, time-of-flight-accessory, photogate, interface

### 5.5 Experimental Procedure



**Figure 5.5** The schematic illustration of experimental setup that includes projectile launcher, time-of-flight-accessory, photogate and plastic ball

The experimental setup of projectile motion is implemented as shown in Figure 5.5. The photogate should be plugged into channel 1 and the time-of-flight accessory should be plugged into channel 2 of the interface. Turn on the interface. Connections between time-of-flight accessory and computer,

photogate and computer will be established through an interface. To collect the data, a software program (Data Studio) will be used.

Here are the procedures for launching Data Studio program to provide data collection and the experimental steps:

- Double-click the Data Studio icon on your desktop to launch the Data Studio software. Introduce photogate to the digital channel 1 and time-of-flight accessory to the digital channel 2 of the interface. And now, the computer is ready to measure the time of flight ( $t_{AD}$ ).
- You can open the table for “time of flight” from the menu on the left-hand side.
- The photogate should be attached to the launcher. Keep the launcher as close to the photogate as possible without interrupting the photogate to make the initial velocity of the ball more accurate.
- The angle of inclination above the horizontal is adjusted by loosening the two thumbscrews and rotating the Launcher barrel to the desired angle. Use the plumb bob and the protractor on the label to select the angle. Tighten both thumbscrews when the angle is set. Record the value of the adjusted angle in Measurements and Results section.
- Place a ball in the muzzle of the Launcher. NOTE: Always cock the piston with a ball in the piston. You may damage the piston if you use the plastic rod without a ball in the piston.
- While looking through the range-setting slots on the top side of the Launcher, push the ball down the barrel with the plastic rod until the trigger catches the edge of the piston at the desired range setting. (The trigger will “click” into place.)
- The ball should be placed in the launcher's SHORT-RANGE stage. When the yellow indicator tape on the piston is visible in the middle range-setting slot, the piston is in the SHORT-RANGE position.
- Remove the plastic rod after you placed the ball.
- When the Projectile Launcher is loaded, the yellow indicator tape is visible through the range-setting slots on the upper side of the barrel. Do not look into the launcher or stay in front of it. To check whether the Launcher is loaded, look through the range-setting slots on the barrel.
- Before shooting the ball, make certain that you are not firing at another person or at anything breakable like glass windows, doors, etc.
- Fire one shot to locate where the ball hits the time-of-flight accessory.
- In order to measure the flight time, the ball must touch a point in the white area on the time-of-flight accessory. So, place the time-of-flight accessory far enough away from the projectile launcher.

- To shoot the ball, pull straight up on the trigger string that is attached to the trigger. You only need to pull about one centimeter.
- The trigger will automatically return to its initial position after you release the string.
- To acquire data, click “Start” on the Data studio program and fire one shot. To stop data collection, click “Stop”.
- Time of flight will be the time it takes for the ball to go from the first photogate to the time-of-flight accessory. Read the “elapsed time”, which corresponds to “ $t_{AD}$ ”, from the table on the Data Studio and record the data in Table 5.1.
- Measure the range  $R$  along the floor from the release point of the ball to the landing point and record your measurement in Table 5.1.
- Repeat the procedure four more times.
- Measure the vertical distance ( $h$ ) from the bottom of the ball as it leaves the barrel to the floor. Record this distance in Measurements and Results section. Note that the position of the ball just before the ball leaves the launcher is marked on the launcher.
- Using the measured horizontal range  $R$ , time-of-flight  $t_{AD}$  and adjusted angle value, calculate the components of the initial velocity and itself for each run. Check the equations derived in the Theoretical Concepts section.
- Then calculate the maximum height that ball reached, climbing time to the maximum height, final projection angle of the projectile, the  $y$ - component of the final velocity and itself. Record the obtained values in Table 5.2 for each run.

### 5.6 Measurements and Results

$h = \dots\dots\dots$

$\theta_0 = \dots\dots\dots$

**Table 5.1** Projectile motion experiment results

Measurement Number	$t_{AD}$ (s)	X (horizontal range) (cm)
1		
2		
3		
4		
5		
6		



7		
8		
9		
10		
Mean Value		
Error		

$$t = (\dots \pm \dots) \text{ s}$$

$$X_{(\text{horizontal range})} = (\dots \pm \dots) \text{ m}$$

**Table 5.2** Calculated results for projectile motion

Measurement Number	$v_{0x}$ (m/s)	$v_{0y}$ (m/s)	$v_0$ (m/s)	$t_{AB}$ (s)	$y_{max}$ (m)	$v_{y(\text{final})}$ (m/s)	$v_{\text{final}}$ (m/s)	$\theta$
1								
2								
3								
4								
5								
6								
7								
8								
9								

10								
Mean Value								
Error								

$$v_{0x} = (\dots \pm \dots) \text{ m/s}$$

$$v_{0y} = (\dots \pm \dots) \text{ m/s}$$

$$v_0 = (\dots \pm \dots) \text{ m/s}$$

$$t_{AB} = (\dots \pm \dots) \text{ s}$$

$$y_{maks} = (\dots \pm \dots) \text{ m}$$

$$v_{y(final)} = (\dots \pm \dots) \text{ m/s}$$

$$v_{final} = (\dots \pm \dots) \text{ m/s}$$

$$\theta = (\dots \pm \dots)^\circ$$

### 5.7 Questions

1. When  $\alpha = 0^\circ$  and  $\alpha = 90^\circ$  explain the situations using equations which is derived for the motion as in Figure 5.3 (Equations (5.14)-(5.20)).
2. When will the velocity take a minimum and maximum value? What does it mean if the y-component of the velocity is negative?
3. Derive the expression required to obtain the time ( $t_{CD}$ ) required for the ball to take the distance CD (as in Figure 5.3) in the experiment.



# Experiment 5

## The Principle of Inertia (Incline)

DEU Faculty of Science

Physics Department

2020

## 6.1 Preparation Questions

4. Explain the Newton's laws of motion briefly.
5. What does the friction force depend on? Explain briefly.
6. What is the unit of coefficient of friction?

## 6.2 Aims of the Experiment

Calculation of the static friction coefficient between the object and the inclined plane through Newton's First Law of Motion

## 6.3 Theoretical Concepts

The relationship between force and motion was first proposed by Isaac Newton (1642-1727). Newton explained the basic principles of dynamics with three expressions in his book "Mathematical Principles of Natural Philosophy" published in 1687. These three statements have been named as Newton's laws of motion.

- Newton's First Law (Principle of Inertia)
- Newton's Second Law (Fundamental Law of Dynamics "F = ma")
- Newton's Third Law (Action-Reaction Principle)

According to Newton's first law, which is the subject of this experiment;

**“Unless a net external force (resultant force) acts on an object, the speed of the object does not change (there is no change in direction, and magnitude), in other words, the object does not accelerate. If the object is stationary, it will remain at rest; if it is moving, it will continue its smooth linear motion with a constant speed.”**

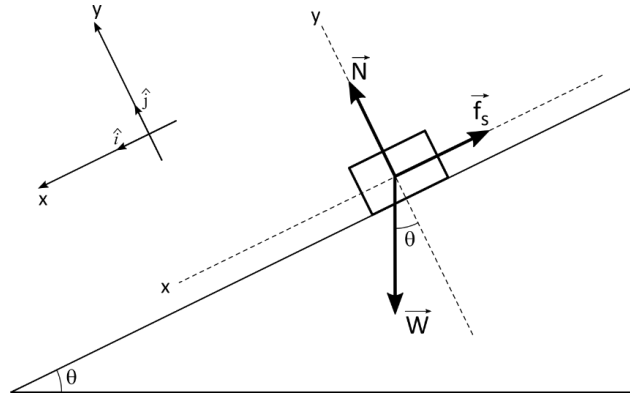
The tendency of an object to resist on changing of its speed (the state of resistance) is called the inertia of that object, and the quantity of this resistance is called the (inertia) mass of the body. When the object is at rest or moving with a constant speed, we define the object is in equilibrium. In order to be in an equilibrium situation, the body must not be subjected to any force or the vectorial sum of applied forces (net force) must be zero;

$$\sum \vec{F} = 0 \quad (6.1)$$

Since total vectorial sum of the forces is zero, each component of the net force must also satisfy the equation;

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0 \quad (6.2)$$

If a block of mass  $m$  on an incline with an angle  $\theta$  is in equilibrium, the vectorial sum of forces acting on the block, which are normal force  $\vec{N}$ , downward gravity force  $\vec{W}$  and static friction force  $\vec{f}_s$ , should be zero as shown in the figure. In this case the equations of motion of the block can be written as;



**Figure 6.1** Free body diagram of an object in inclined plane with friction

$$\begin{aligned}\sum \vec{F} &= m\vec{a} \quad (a = 0) \\ \vec{N} + \vec{W} + \vec{f}_s &= 0 \\ N\hat{j} - mg \cos \theta \hat{j} + mg \sin \theta \hat{i} - f_s \hat{i} &= 0 \\ \sum F_x &= mg \sin \theta - f_s = ma_x = 0 \\ \sum F_y &= N - mg \cos \theta = ma_y = 0\end{aligned}\tag{6.3}$$

#### 6.4 Tools to be used in the experiment

- Statics board
- Inclined plane
- Pulley
- Wooden Block
- Mass and hanger set
- Thread

#### 6.5 Experimental Procedure

1. Mount the inclined plane on the statics board. Set the wooden block on the inclined plane and use thread to connect it over a pulley to a mass hanger.

2. **First**, set the angle as  $\theta = 0$ . Carefully adjust the pulley so that the thread is parallel to the inclined plane. Add or subtract masses on the mass hanger to determine balance state of the block just before it moves.
3. After obtaining the balance state, measure the mass of block  $m_B$  and the hanging mass (total mass of the mass hanger and added masses). The mass of the mass hanger is 5g  $m$  and record the Table 6.1.
4. Calculate the weight of the block  $W_B$  and added masses  $W$ , and record the data Table 6.1.
5. Draw the free-body diagram for both objects (block and hanging masses) and write the corresponding equations using Newton's First Law of Motion.

*For*

$m_B$ .....  
 .....

*For*

$m$ .....  
 .....

6. Analyse the equations in order to calculate the static friction coefficient ( $\mu_s$ )

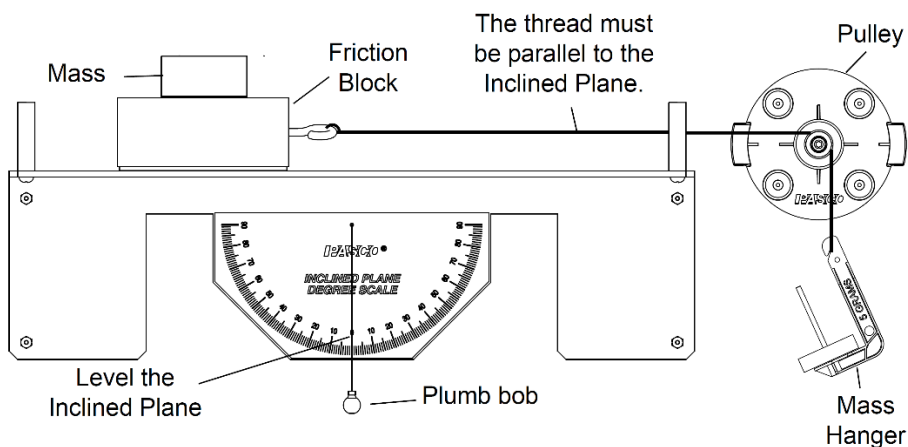


Figure 6.2 Equipment setup for the first stage of the experiment

7. **Second**, set the angle as  $\theta \neq 0$ . Bring the system into balance by changing the angle  $\theta$  in order to determine the *critical angle* at which the block can stand on the inclined plane without slipping.
8. Draw the free-body diagram for the block and write the corresponding equations using Newton's First Law of Motion.

*For*

$m_B$ .....  
.....

*For*

$m$ .....  
.....

9. Analyse the equations in order to calculate the static friction coefficient ( $\mu_s$ )



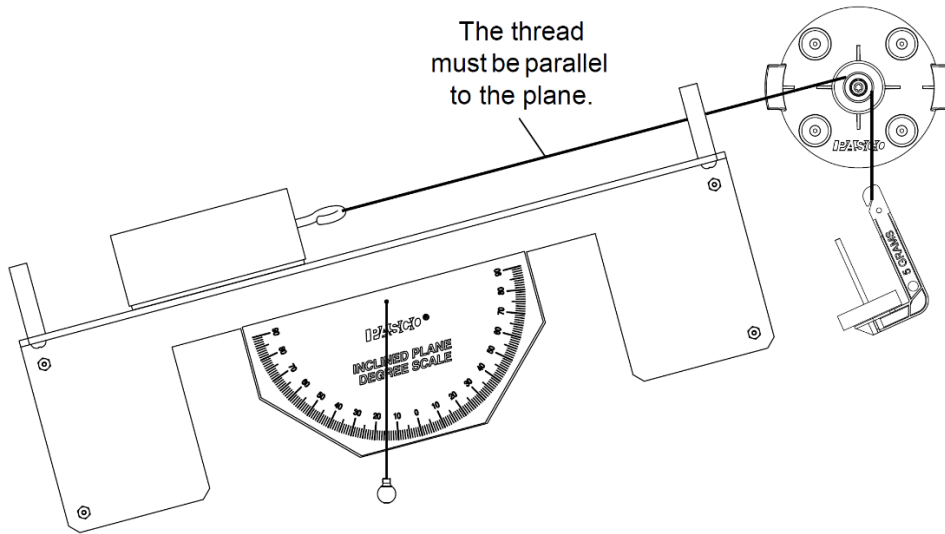


Figure 6.3 Equipment setup for the second stage of the experiment

## 6.6 Measurements and Results

Table 6.1 Results of measurements

	$\theta$ ( $^\circ$ )	$m$ (kg)	$W$ (N)	$m_B$ (kg)	$W_B$ (N)	$f_s$ (N)	$\mu_s$
1. stage							
2. stage							

10. You can calculate the uncertainty in determining the static friction coefficient with the following equations;

$\mu_s = f(m, m_B, \theta)$ $\Delta\mu_s = \sqrt{\left(\frac{\partial\mu_s}{\partial m}\Delta m\right)^2 + \left(\frac{\partial\mu_s}{\partial m_B}\Delta m_B\right)^2 + \left(\frac{\partial\mu_s}{\partial\theta}\Delta\theta\right)^2}$ $\mu_s = \bar{\mu}_s \pm \Delta\mu_s$	(6.4)
--	-------

## 6.7 Questions

4. How much weight should be hung on the mass hanger to keep the system balanced when the inclination angle is  $60^\circ$ ?
5. In what situations is the friction force a necessary force and in which situations it must be eliminated?

6. What can be done to reduce the friction coefficient between two surfaces?



## **Experiment 6**

# **Newton's Second Law Work-Kinetic Energy Theorem**

**DEU Faculty of Science**

**Physics Department**

2020

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## 7.1 Preparation Questions

- 1) Explain the Newton's Second law briefly.
- 2) Could the work be zero when the force and displacement are not zero? Explain briefly.

## 7.2 Aims of the Experiment

Determine the relationship between an object's or system's mass, acceleration, and the net force applied to the object or system experimentally.

Investigate the relationship between the change in kinetic energy of an object experiencing a non-zero net force, and the work done by that net force on the object.

## 7.2 Newton's Second Law

Newton's laws, the basis of classical mechanics, are used to explain a wide range of physical phenomena. For example, the motions of stars and planets obey Newton's laws. However, it is important to note that these laws are invalid under two conditions.

1. In cases where the velocities of the objects are close to the speed of light (cases where the velocity of the object is greater than approximately  $0.1c$ ). For such cases, it is required to use the special theory of relativity developed by Einstein.
2. In cases where the dimensions of the object under investigation are very small (electron, proton, neutron, etc.). It is required to use Quantum Physics for such sub-atomic particle's world.

Newton's first law expresses the tendency of objects to preserve their original status. *Newton's 1<sup>st</sup> Law: In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).* In simpler terms, we can say that when no force acts on an object, the acceleration of the object is zero.

If you want to create a change in the velocity (acceleration) of the object, it is required to apply a force on the object. In other words, acceleration and force are directly proportional ( $a \propto F$ ). The constant of proportionality between force and acceleration can be called inertial mass.

The tendency of an object to resist any attempt to change in its velocity is called *inertia*.

**Mass** is a quantity of an object's resistance against being accelerated by a force. The greater the mass of an object, the less acceleration of it under the influence of applied force.

Suppose a force  $F$  acting on an object of mass  $m_1$  generates an acceleration  $a_1$ , and the same force acting on an object of mass  $m_2$  generates an acceleration  $a_2$ .



**Figure 7.1** The effect of a constant force  $\vec{F}$  on the acceleration of two objects with different mass.

The ratio of the two masses is defined as the inverse ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Accordingly, two objects with different inertia under the influence of the same force will accelerate inversely proportional to their masses. In other words, a greater change will occur in the velocity of an object with less mass under the effect of the same force. Similarly, for the object in motion, the greater the mass, the greater the force is required to stop it.

$m_1 \cdot a_1 = m_2 \cdot a_2 = \dots = \text{cont} \rightarrow a \propto \frac{1}{m} \Rightarrow$  The acceleration is inversely proportional to the mass of the object.



**Figure 7.2** The effect of different forces  $\vec{F}$  applied to two objects of the same mass on the acceleration.

As seen in Figure 7.2, when different forces  $\vec{F}_1, \vec{F}_2, \dots$  are applied to an object of mass  $m$ , the object gains different accelerations  $a_1, a_2, \dots$

The ratio of the force to the acceleration of the objects;

$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \dots = \text{constant} \Rightarrow F \propto a$  Acceleration is directly proportional to the force applied.

If we express this ratio with  $m$ :

$$\frac{F}{a} = m \rightarrow F = m a \quad (7.1)$$

Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it. The mass size is represented by [M] and measured in kilograms (kg) in the SI unit system.

As a result, Newton's second law gives the relationship between the force applied on an object and the mass of the object and the acceleration.

**Newton's 2nd Law: The acceleration of an object is directly proportional to the resultant force (or net force,  $\Sigma F$ ) acting on it, and inversely proportional to its mass.** Generally (for more than one force);

$$\vec{F}_{net} = \sum \vec{F}_i = m\vec{a} \quad (7.2)$$

the equation is called Newton 2nd law or the fundamental law of Dynamics. The SI unit of force is the Newton, and it is indicated by the letter N. Newton is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s<sup>2</sup>.

$$1 N = (1 kg) \cdot (1 m/s^2)$$

### 7.3 Work-Kinetic Energy Theorem

Consider an object that undergoes a displacement of magnitude  $s$  along a straight line. While the object moves, a constant force  $\vec{F}$  acts on it in the same direction as the displacement  $\vec{\Delta s}$ , which can be defined as  $\vec{\Delta s} = \vec{x}_2 - \vec{x}_1 = \vec{\Delta x}$  (Fig. 7.3). We define the **work**  $W$  done by this constant force under these circumstances as the scalar product of the force  $\vec{F}$  and the displacement  $\vec{\Delta x}$ :

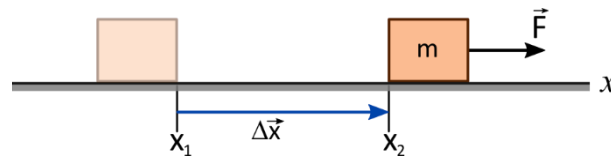
$$W = \vec{F} \cdot \vec{\Delta x} \quad (7.3)$$

Work is a *scalar* quantity, even though it's calculated from two vector quantities (force and displacement). Think of a person pushing a stalled car. If he pushes the car through a displacement  $\vec{\Delta x}$  with a constant force  $\vec{F}$  in the direction of motion, the amount of work he does on the car is given by Eq.  $W = F\Delta x$ . But what if the person pushes at an angle  $\theta$  to the car's displacement. The  $\vec{F}$  has a component  $F_x = F \cos \theta$  in the direction of the displacement  $\vec{\Delta x}$  and a component  $F_y = F \sin \theta$  that acts perpendicular to  $\vec{\Delta x}$ . (Other forces must act on the car so that it moves along  $\vec{\Delta x}$ , not in the direction of  $\vec{F}$ . We're interested in only the work that the person does, however, so we'll consider only the force he exerts.) Only the parallel component is effective in moving the car, so we define the work as the product of this force component and the magnitude of the displacement.

Hence,

$$W = (|\vec{F}| \cos \theta) |\vec{\Delta x}|$$

If  $\theta = 0$ , so that  $\vec{F}$  and  $\Delta\vec{x}$  are in the same direction, then  $\cos \theta = 1$  and we are back to Eq. ( $W = F\Delta x$ ). In the above equation has the form of the *scalar product* of two vectors.



**Figure 7.3** The work done by a constant force acting in the same direction as the displacement.

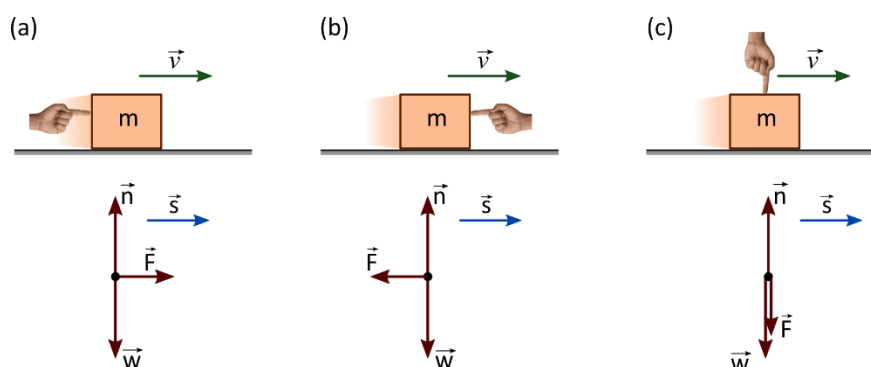
The SI unit of work is the **joule**. The unit of work is the unit of force multiplied by the unit of distance. In SI units the unit of force is the newton and the unit of distance is the meter, so 1 joule is equivalent to 1 *newton-meter* (N.m):

$$1 \text{ Joule} = (1 \text{ Newton}) \cdot (1 \text{ meter}) \text{ or } J = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2 / \text{s}^2$$

If you lift an object with a weight of 1 N a distance of 1 m at a constant speed, you exert a 1 N force on the object in the same direction as its 1 m displacement and so do 1 J of work on it.

It's important to understand that work can also be negative or zero. This is the essential way in which work as defined in physics differs from the "everyday" definition of work. When the force has a component in the *same direction* as the displacement ( $\theta$  between  $0^\circ$  and  $90^\circ$ ),  $\cos\theta$  in the above equation is positive and the work  $W$  is *positive*. When the force has a component *opposite* to the displacement ( $\theta$  between  $90^\circ$  and  $180^\circ$ ),  $\cos\theta$  is negative and the work is *negative*. When the force is *perpendicular* to the displacement,  $\theta = 90^\circ$  and the work one by the force is *zero*.

How do we calculate work when *several* forces act on an object? One way is to compute the work done by each separate force. Then, because work is a scalar quantity, the *total* work  $W_{\text{tot}}$  done on the object by all the forces is the algebraic sum of the quantities of work done by the individual forces. An alternative way to find the total work  $W_{\text{tot}}$  is to compute the vector sum of the forces (that is, the net force) and then use this vector sum as  $\vec{F}$ .



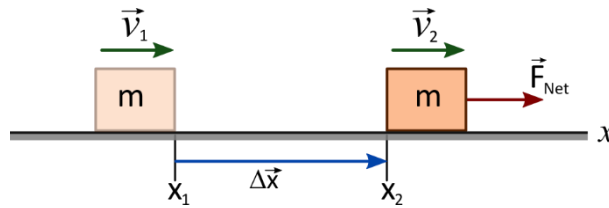
**Figure 7.4** The relationship between the total work done on an object and how the object's speed changes.



The total work done on an object by external forces is related to the object's displacement that is, to changes in its position. But the total work is also related to changes in the *speed* of the object. To see this, consider Fig. 6.4, which shows a block sliding on a frictionless table. The forces acting on the block are its weight  $\vec{w}$ , the normal force  $\vec{n}$ , and the force  $\vec{F}$  exerted on it by the hand.

In Fig. 7.4(a) the net force on the block is in the direction of its motion. From Newton's second law, this means that the block speeds up, this also means that the total work  $W_{tot}$  done on the block is positive. The total work is *negative* in Fig. 7.4(b) because the net force opposes the displacement; in this case the block slows down. The net force is zero in Fig. 7.4(c), so the speed of the block stays the same and the total work done on the block is zero. We can conclude that *when a particle undergoes a displacement, it speeds up if  $W_{tot} > 0$ , slows down if  $W_{tot} < 0$ , and maintains the same speed if  $W_{tot}=0$ .*

Let's make this more quantitative. In Fig. 7.5, a particle with mass  $m$  moves along the  $x$ -axis under the action of a constant net force with magnitude  $F$  that points in the positive  $x$ -direction. The particle's acceleration is constant and given by Newton's second law ( $F = m \cdot a_x$ ).



**Figure 7.5** A constant net force  $\vec{F}$  does work on a moving object.

As the particle moves from point  $x_1$  to  $x_2$ , it undergoes a displacement  $\vec{\Delta x} = x_2 - x_1$  and its speed changes from  $v_1$  to  $v_2$ .

$$v_2^2 = v_1^2 + 2\vec{a} \cdot \vec{\Delta x} \quad \vec{a} \cdot \vec{\Delta x} = \frac{v_2^2 - v_1^2}{2}$$

$$m\vec{a} \cdot \vec{\Delta x} = m \frac{v_2^2 - v_1^2}{2} \Rightarrow \vec{F} \cdot \vec{\Delta x} = m \frac{v_2^2 - v_1^2}{2}$$

Hence, the expression  $W = \vec{F} \cdot \vec{\Delta x} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$  is obtained.

In this equation the product  $\vec{F} \cdot \vec{\Delta x}$  is the work done by the net force  $\vec{F}$  and thus is equal to the total work  $W_{tot}$  done by all the forces acting on the particle. The quantity  $\frac{1}{2}mv^2$  is called the **kinetic energy**

$K$  of the particle:  $K = \frac{1}{2}mv^2$

Like work, the kinetic energy of a particle is a scalar quantity; it depends on only the particle's mass and speed, not its direction of motion. Kinetic energy can never be negative, and it is zero only when the particle is at rest.

We can now interpret in above equation in terms of work and kinetic energy. The first term on the right side of equation is  $K_2 = \frac{1}{2}mv_2^2$ , the final kinetic energy of the particle (that is, after the displacement). The second term is the initial kinetic energy,  $K_1 = \frac{1}{2}mv_1^2$ , and the difference between these terms is the *change* in kinetic energy. So equation says:

$$W_{Tot} = K_2 - K_1 = \Delta K \quad (7.4)$$

This **work–energy theorem** agrees with our observations about the block in Fig. 7.4. When  $W_{tot}$  is *positive*, the kinetic energy *increases* (the final kinetic energy  $K_2$  is greater than the initial kinetic energy  $K_1$ ) and the particle is going faster at the end of the displacement than at the beginning. When  $W_{tot}$  is *negative*, the kinetic energy *decreases* ( $K_2$  is less than  $K_1$ ) and the speed is less after the displacement. When  $W_{tot} = 0$ , the kinetic energy stays the same ( $K_1 = K_2$ ) and the speed is unchanged. Note that the work–energy theorem by itself tells us only about changes in *speed*, not *velocity*, since the kinetic energy doesn't depend on the direction of motion.

Because we used Newton's laws in deriving the work–energy theorem, we can use this theorem only in an inertial frame of reference. Note that the work–energy theorem is valid in *any* inertial frame, but the values of  $W_{tot}$  and  $K_2 - K_1$  may differ from one inertial frame to another (because the displacement and speed of an object may be different in different frames).

To accelerate a particle of mass  $m$  from rest (zero kinetic energy) up to a speed  $v$ , the total work done on it must equal the change in kinetic energy from zero to  $K_2 = \frac{1}{2}mv^2$ :

$$W_{Tot} = K - 0 = K = \frac{1}{2}mv^2$$

*So the kinetic energy of a particle is equal to the total work that was done to accelerate it from rest to its present speed.*

Energy associated with position is called **potential energy**. The potential energy associated with an object's weight and its height above the ground is called *gravitational potential energy*. We now have *two* ways to describe what happens when an object falls without air resistance. One way, is to say that a falling object's kinetic energy increases because the force of the earth's gravity does work on the object. The other way is to say that the kinetic energy increases as the gravitational potential energy decreases.

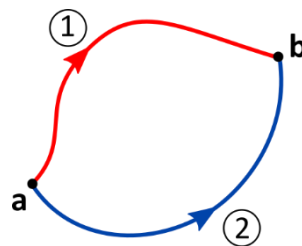
We want to find the work done by the weight when the object moves downward from a height  $y_1$  above the origin to a lower height  $y_2$ . The weight and displacement are in the same direction, so the work  $W_{grav}$  done on the object by its weight is positive:

$$W_{grav} = \vec{F} \cdot \vec{\Delta s} = F \cdot \Delta s \cdot \cos(0) = w(y_1 - y_2) = mgy_1 - mgy_2 = -\Delta U_g$$

This expression also gives the correct work when the object moves *upward* and  $y_2$  is greater than  $y_1$ . In that case the quantity  $(y_1 - y_2)$  is negative, and  $W_{grav}$  is negative because the weight and displacement are opposite in direction. Equation shows that we can express  $W_{grav}$  in terms of the values of the quantity  $mgy$  at the beginning and end of the displacement. This quantity is called the **gravitational potential energy**,  $U_{grav}$  :

$$U_{grav} = mgh \quad (7.5)$$

The unit of potential energy is the joule (J), the same unit as is used for work.



**Figure 7.6** The work done by a conservative force such as gravity depends on only the endpoints of a path, not the specific path taken between those points.

**Conservative and Nonconservative Forces:** When you throw a ball in the air, it slows down as kinetic energy is converted to gravitational potential energy. But on the way down the ball speeds up as potential energy is converted back to kinetic energy. If there is no air resistance, the ball is moving just as fast when you catch it as when you threw it. Another example is a glider moving on a frictionless horizontal air track that runs into a spring bumper. The glider compresses the spring and then bounces back. If there is no friction, the glider ends up with the same speed and kinetic energy it had before the collision. Again, there is a two-way conversion from kinetic to potential energy and back. In both cases the total mechanical energy, kinetic plus potential, is constant or *conserved* during the motion.

A force that offers this opportunity of two-way conversion between kinetic and potential energies is called a **conservative force**. We have seen two examples of conservative forces: the gravitational force and the spring force. An essential feature of conservative forces is that their work is always *reversible*. Anything that we deposit in the energy “bank” can later be withdrawn without loss. Another important aspect of conservative

forces is that if an object follows different paths from point 1 to point 2, the work done by a conservative force is the same for all of these paths (Fig. 7.6).

Not all forces are conservative. Consider the friction force acting on the crate sliding on a ramp. When the crate slides up and then back down to the starting point, the total work done on it by the friction

force is *not* zero. When the direction of motion reverses, so does the friction force, and friction does *negative* work in *both* directions. Friction also acts when a car with its brakes locked skids with decreasing speed (and decreasing kinetic energy). The lost kinetic energy can't be recovered by reversing the motion or in any other way, and total mechanical energy is *not* conserved. So there is *no* potential-energy function for the friction force.

In the same way, the force of fluid resistance is not conservative. If you throw a ball up in the air, air resistance does negative work on the ball while it's rising *and* while it's descending. The ball returns to your hand with less speed and less kinetic energy than when it left, and there is no way to get back the lost mechanical energy. A force that is not conservative is called a **nonconservative force**. The work done by a nonconservative force *cannot* be represented by a potential-energy function.

Nonconservative forces cannot be represented in terms of potential energy. But we can describe the effects of these forces in terms of kinds of energy other than kinetic or potential energy. When a car with locked brakes skids to a stop, both the tires and the road surface become hotter. The energy associated with this change in the state of the materials is called internal energy. Raising the temperature of an object increases its internal energy; lowering the object's temperature decreases its internal energy.

$$\Delta E = \Delta K + \Delta U = 0 \quad (7.6)$$

This remarkable statement is the general form of the **law of conservation of energy**. In a given process, the kinetic energy, potential energy, and internal energy of a system may all change. But the *sum* of those changes is always zero. If there is a decrease in one form of energy, it is made up for by an increase in the other forms. When we expand our definition of energy to include internal energy, in the above equation says: *Energy is never created or destroyed; it only changes form*. No exception to this rule has ever been found.

$$\Delta K = -\Delta U = W$$

In this case, the relationship between work and potential energy can be written as,

$$W = -\Delta U$$

Thus, the relationship between a conservative force and potential energy can be expressed as,

$$\vec{F}_{\text{korunumlu}} = -\vec{\nabla}U(x, y, z) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k} \quad (7.7)$$

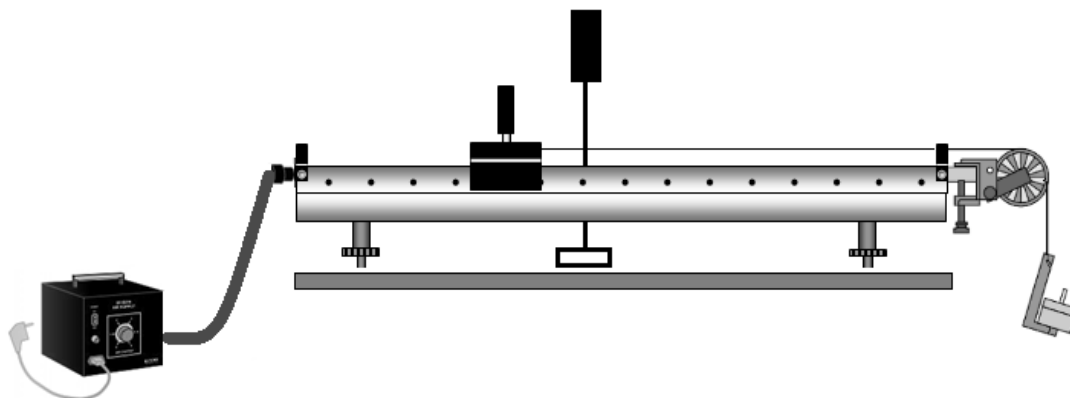
$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

## 7.4 Tools to be used in the experiment

- Air Track (The Air Rail provides almost frictionless movement. Air rails are used to minimize the friction force as much as possible.)
- Air Supply with Hose
- Glider, Photogate Flag
- Mass hanger, mass set, string
- Pulleys with clamps
- Photogate
- Computer, Interface and Data Studio software

## 7.5 Experimental Procedure

### 7.5.1 Newton's Second Law



**Figure 7.7** Newton's Second Law Experiment Setup

- Place the air track on a flat surface. Start the air supply.
- Be sure the air track is as level as possible. For this, place the glider that you will use in the experiment in the middle of the air track. **(Do not move the glider on the track while the air supply is turned off!)**
- Control if the glider moves on the air track when you turn on the air supply. If the glider moves continuously in one direction, it means there is a slope. To eliminate this slope, adjust the screwed feet under the air track accordingly.
- Set up the experimental setup as shown in Figure 7.7. With the help of a string that will pass through the pulley, connect the glider to the mass hanger. Adjust the angle of the pulley so that the string is parallel to the air track.

- Measure the diameter ( $x$ ) of the Photogate Flag on the glider.
- Run "Data Studio" program on the computer. After selecting the channel (1 or 2) that the photogate is connected to, " Choose sensor or instrument " section will open. Select the " Photogate " section from here.
- Keep the glider as close to the photogate as possible without interrupting the photogate to make the initial velocity of the glider more accurate. Click the "Start" button to begin collecting data.
- Determine how long it takes the glider to travel the distance  $x$  with the help of the photogate and datastudio program. Note the measurement values in the table.
- Measure the value of hanging mass  $m$  with the help of a balance. Note the measurement values in the table.
- Write Newton's 2nd Law by showing the forces affecting the hanging mass  $m$  on the free body diagram. Find the tension in the string,  $T$ .
- By showing the forces affecting the mass  $M$  of the glider on the free body diagram, write Newton's 2nd Law and find the value of the mass  $M$ .
- Measure the mass of the glider with the help of a balance. Note the measurement values in the table.
- Compare the value you found for the mass ( $M$ ) of the glider with the value you found with the precision balance.
- Calculate the relative error.

### 7.5.2 Work-Energy Theorem

- In the first part of the test, when the system consisting of masses  $m$  and  $M$  was released from rest, a work is done by the tensile force of the string on mass  $M$ . Since this force causes the mass of  $M$  to displacement of  $x$ , calculate the work done by the tensile force  $T$  on mass  $M$ , which you found in the first part of the experiment.

$$W = \dots\dots\dots$$

- Calculate the change in kinetic energy on the glider with mass  $M$  starting from rest ( $v_1 = 0$ ) and having a velocity  $v_2$  after  $t$  time, by using below equation:

$$\Delta K = \frac{1}{2} M v_2^2 - \frac{1}{2} M v_1^2$$

$$\Delta K = \dots\dots\dots$$

- Compare your results.

### 7.6 Measurements and Results

$$x = (\dots\dots\dots \pm \dots\dots\dots) \text{ m} \quad M = (\dots\dots\dots \pm \dots\dots\dots) \text{ kg}$$

**Table 7.1** Newton's Second Law experimental results

Measurement Number	t (s)
1	
2	
3	
4	
5	
<b>Mean</b>	
<b>Error</b>	

$$t = (\dots\dots\dots \pm \dots\dots\dots) \text{ s}$$

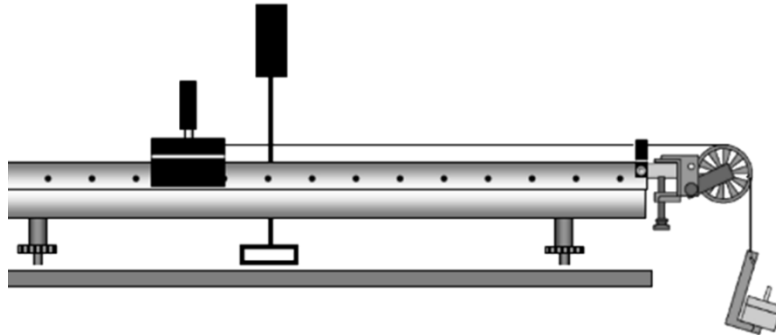
**Table 7.2** Newton's Second Law calculated Results

<b><i>a</i> (m/s<sup>2</sup>)</b>	<b><i>T</i> (N)</b>	<b><i>M</i> (kg)</b>
±	±	±

$$M_{\text{Measurement}} = (\dots\dots\dots \pm \dots\dots\dots) \text{ kg}$$

**Table 7.3** Calculation results for the work-energy relationship.

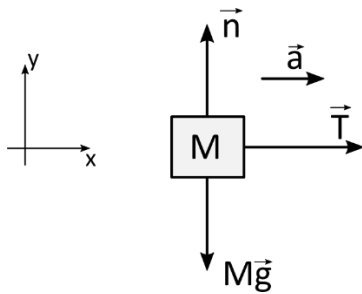
<b><i>v</i><sub>2</sub> (m/s)</b>	<b><math>\Delta K</math> (J)</b>	<b><i>W</i> (J)</b>
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**Figure 7.8** Free body diagram of the forces acting on the sled of mass M and mass m.

For sled with mass M;

Free-Body Diagram



Newton's Second Law

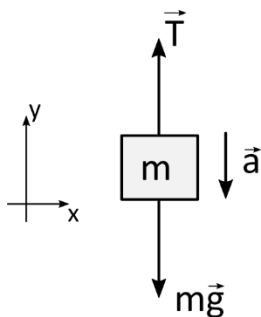
$$\sum \vec{F} = M\vec{a}$$

$$x \rightarrow T = Ma$$

$$y \rightarrow n - Mg = 0$$

for mass m;

Free-Body Diagram



Newton's Second Law

$$\sum \vec{F} = M\vec{a}$$

$$y \rightarrow T - mg = -ma$$

$$T - mg = m(-a)$$

$$Ma - mg = m(-a)$$



$$M = \frac{m(g - a)}{a}$$

$$T = m(g - a)$$

### 7.7 Questions

1. What can you say about the Work-Kinetic Energy relationship according to your results in section II of the experiment? Does the Work-Energy theorem depend on whether the force applied to the system is constant or variable? Explain.
3. Show all the forces in the system on the figure.
4. Does the principle of conservation of energy also apply to other types of energy other than mechanical energy? Discuss.
5. If an object is at rest, can you say that there are no external forces acting on it? Explain.



# Experiment 7

## Collisions in One Dimension

DEU Faculty of Science

Physics Department

2020

## 8.1 Preparation Questions

1. Explain the momentum concept. What does conservation of momentum mean?
2. For each of the following situations, state the types of collisions that are elastic, inelastic or completely inelastic, with reasons.
  - When you release an elastic ball, it hits the ground and bounces and reaches your hand again.
  - When you release another ball from your hand, it bounces off the ground, but reaches only half the height it was released.
  - When you let go of a clay ball, it falls to the ground and stops.

## 8.2 Aim of the Experiment

Investigation of the conservation principles of momentum and kinetic energy for different types of collisions made by two bodies in linear motion.

## 8.3 Linear Momentum

The linear momentum of a particle or an object that can be modeled as a particle of mass  $m$  moving with a velocity  $\mathbf{v}$  is defined to be the product of the mass and velocity;

$$\vec{p} = m\vec{v} \quad (8.1)$$

The linear momentum is a vector quantity because it equals the product of a scalar quantity  $m$  and a vector quantity  $\mathbf{v}$ . Its direction is along  $\mathbf{v}$ , it has dimensions  $ML/T$ , and its SI unit is  $kg\ m/s$ . If a particle is moving in an arbitrary direction,  $\vec{p}$  must have three components;

$$p_x = mv_x \quad p_y = mv_y \quad p_z = mv_z \quad (8.2)$$

Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle. We start with Newton's second law and substitute the definition of acceleration;

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt} \quad (8.3)$$

If the mass of the system acted under the force  $\vec{F}_{net}$  does not change ( $\frac{dm}{dt} = 0$ ), the above expression is reduced to the form  $\vec{F}_{net} = m \frac{d\vec{v}}{dt} = m\vec{a}$ . In this case, we can generally define Newton's II law (net force  $\vec{F}_{net} = m\vec{a}$ ) as the change of the momentum of the system with time in a more inclusive

expression. If the momentum of the particle changes from  $\vec{p}_i$  at time  $t_i$  to  $\vec{p}_f$  at time  $t_f$ , integrating Equation 8.3 gives;

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F} dt \quad (8.4)$$

To evaluate the integral, we need to know how the force varies with time. The quantity on the right side of this equation is called the **impulse** of the force  $\vec{F}$  acting on a particle over the time interval  $\Delta t = t_f - t_i$ . Impulse is a vector defined by  $\vec{I}$ .

**Conservation of linear momentum principle:** As it is known, the basic quantities that define particles in classical physics are *charge* and *mass*. The total momentum of a system consisting of two particles of mass  $m_1$  and  $m_2$  without charge is always conserved. When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. This is depicted by the following expression;

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad (8.5)$$

**Collision** can be considered as a short-term interaction between two bodies or more objects that causes changes in the motion positions of the objects due to the internal forces. These interaction forces arise from the electrostatic interaction of electrons in the surface atoms of two bodies. The total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision. In contrast, the total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether kinetic energy is conserved is used to classify collisions as either elastic or inelastic.

**Elastic Collisions**, an elastic collision between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision. Collisions between certain objects in the macroscopic world, such as billiard balls, are only approximately elastic because some deformation and loss of kinetic energy take place.

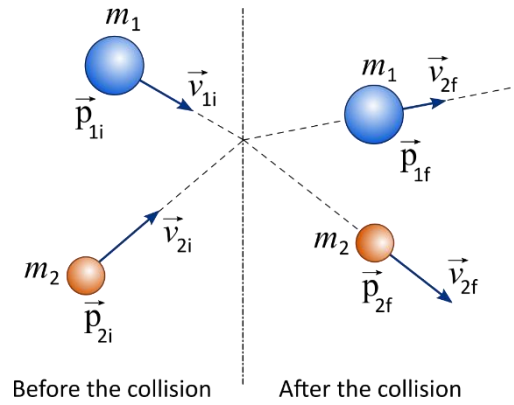
**Inelastic Collision**, an inelastic collision is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved.) In these collisions, the objects move independently from each other after the collision. Two vehicles moving in different directions after colliding, a bullet piercing a wedge, can be given as an example.

**Perfectly Inelastic Collisions**, kinetic energy decreases as in an inelastic collision. In this collision, the objects are interlocked and continue to move together after the collision. Two vehicles moving

together after colliding, a bullet getting stuck in a wedge, proton and electron colliding to form hydrogen atom can be given as examples.

### Elastic Collision

Both momentum and kinetic energy are conserved for two particles that collide as in Figure 8.1.



**Figure. 8.1** Elastic Collision

Conservation of momentum; 
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \tag{8.6}$$

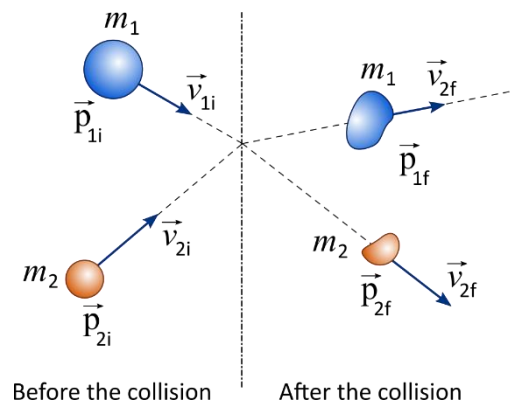
$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Conservation of kinetic energy; 
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \tag{8.7}$$

We shall indicate  $\vec{v}$  as positive if a particle moves to the right and negative if it moves to the left.

### Inelastic Collision

Kinetic energy is not conserved in an inelastic collision. In other words, there is a loss of kinetic energy in this type of collision. The total kinetic energy after the collision is smaller than the total kinetic energy before the collision. The energy difference between the total kinetic energies before and after the collision is either converted into heat energy or stored as potential energy in colliding objects (Figure 8.2).

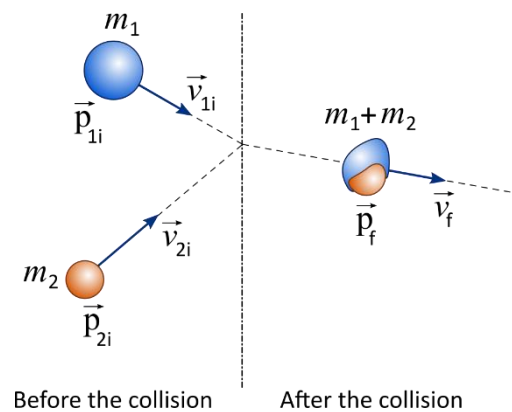


**Figure 8.2** Inelastic Collision

$$\begin{aligned}\vec{p}_{1i} + \vec{p}_{2i} &= \vec{p}_{1f} + \vec{p}_{2f} \\ m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f}\end{aligned}\quad (8.8)$$

### Perfectly Inelastic Collisions

Consider two particles of masses  $m_1$  and  $m_2$  moving with initial velocities  $\vec{v}_{1i}$  and  $\vec{v}_{2i}$  along the same straight line, as shown in Figure 8.3. The two particles collide head-on, stick together, and then move with some common velocity  $\vec{v}_f$  after the collision. Because the momentum of an isolated system is conserved in any collision, we can say that the total momentum before the collision equals the total momentum of the composite system after the collision.


**Figure 8.3** Perfectly Inelastic Collision

$$\begin{aligned}\vec{p}_{1i} + \vec{p}_{2i} &= \vec{p}_f \\ m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= (m_1 + m_2)\vec{v}_f\end{aligned}\quad (8.9)$$

### 8.4 Tools to be used in the experiment

Air Track, Air Supply with Hose, Photogate and Flag, Precision balance, Glider, Ruler, Air Track Accessory kit: Bumper, Needle and wax tube, Computer, Interface and Data Studio software.

### 8.5 Experimental Procedure

1. Place the air track on a flat surface. Be sure the air track is as level as possible. For this, place the glider that you will use in the experiment in the middle of the air track. Control if the glider moves on the air track when you turn on the air supply. If the glider moves continuously in one direction, it means there is a slope. To eliminate this slope, adjust the screwed feet under the air track accordingly.

2. Make connections with the computer by placing the photogate as shown in the figure.

### Part 1: Elastic Collision

3. Attach bumper to the gliders.
4. Measure the diameter ( $x$ ) of the Photogate Flag on the glider. Note the measurement values in the table 8.1.
5. Measure the mass of the glider (with all parts attached) with the help of a balance. Note the measurement values in the table 8.1.
6. Run "Data Studio" program on the computer. After selecting the channel (1 and 2) that the photogate is connected to, " Choose sensor or instrument " section will open. Select the " Photogate " section from here.
7. Start the Air Supply and place the glider in their positions in Figure 8.4. Start the DataStudio program. Then push the glides with a photogate with your hand slightly and watch for collisions in the collision zone located in the middle of the photogates.
8. After the gliders made an elastic collision, they will pass through the photogate for a second time, moving in the opposite direction of their first movement direction. Stop "Data Studio" program after the gliders pass through the photogate for the second time.
9. Open the Tables for both photogates from the Table tab. (Time at door channel 1 and time at door channel 2). In the tables, you will get values two time because the glides pass through the photogate twice. Record these values in Table 8.1.
10. Calculate the velocity values of the masses before and after the collision.
11. Determine whether momentum is conserved or not by calculating the momentum before and after the collision.
12. Determine whether kinetic energy is conserved or not by calculating the kinetic energy before and after the collision.

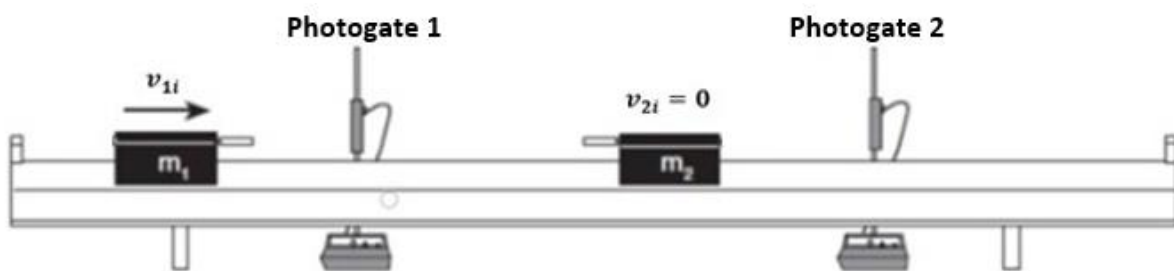




**Figure 8.4** Elastic Collision

**Part 2: Inelastic Collision**

13. Start the Air Supply and place the gliders in the positions in Figure 8.5 this time. Attach needle and wax tube to the gliders. Start the DataStudio program. Then, push the glider with a Photogate with your hand slightly and observe that the gliders stick together by hitting the other glider in the collision area located in the middle of the photogate.
14. Stop the DataStudio program after the gliders move together and pass through the second photogate.
15. Open the Tables for both photogates from the Table tab (time channel 1 and time channel 2). Record the time data you read from the tables in Table 8.2.
16. Calculate the velocity values of the masses before and after the collision.
17. Determine whether momentum is conserved or not by calculating the momentum before and after the collision.
18. Determine whether kinetic energy is conserved or not by calculating the kinetic energy before and after the collision.


**Figure 8.5** Inelastic Collision

## 8.6 Measurements and Results

**Table 8.1** Measurement and calculation results for elastic collision

	$m$ (kg)	$x$ (m)	$t_{initial}$ (s)	$t_{final}$ (s)	$v_{initial}$ (m/s)	$v_{final}$ (m/s)
<b>1. Mass</b>						
<b>2. Mass</b>						

	$p_{initial}$ (kgm/s)	$p_{final}$ (kgm/s)	$E_{initial}$ (J)	$E_{final}$ (J)
<b>1. Mass</b>				
<b>2. Mass</b>				
<b>TOTAL</b>				

**Table 8.2** Measurement and calculation results for inelastic collision

	$m$ (kg)	$x$ (m)	$t_{initial}$ (s)	$t_{final}$ (s)	$v_{initial}$ (m/s)	$v_{initial}$ (m/s)
<b>1. Mass</b>						
<b>2. Mass</b>						

	$p_{initial}$ (kgm/s)	$p_{final}$ (kgm/s)	$E_{initial}$ (J)	$E_{final}$ (J)
<b>1. Mass</b>				
<b>2. Mass</b>				

TOTAL				
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### 8.7 Questions

1. For each type of collision, compare the vector sum of the momentums after the collision with the initial momentum before the collision. Explain.
2. Is kinetic energy conserved for all types of collisions you have done in the experiment? Explain with the reasons?
3. In a situation where we cannot neglect the friction in the experimental setup and the environment, which of the two experiments in which the gliders are pushed towards each other from relatively close and far distances; could be done with fewer errors provided all other conditions were the same? Why?
4. What are your sources of error in the experiment?



## **Experiment 8**

# **Uniform Circular Motion (Centripetal Force)**

**DEÜ Science Faculty  
Physics Department**

2020

## 9.1 Pre-experiment Questions

- 1) Explain the concepts of frequency, period, angular frequency, linear velocity and angular velocity.
- 2) What is the Centripetal Force? Explain.
- 3) Explain the uniform circular motion.

## 9.2 Aims of the Experiment

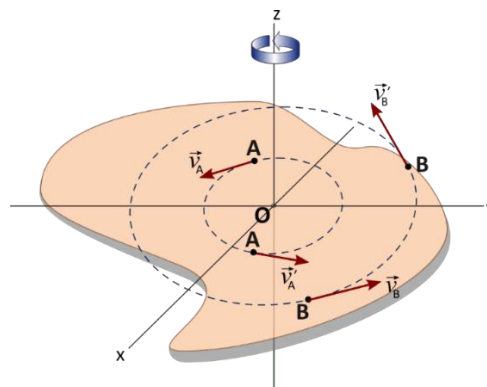
Studying the motion of an object making uniform circular motion and calculating the centripetal force

## 9.3 Circular Motion

Rotational motion of objects around a fixed axis is defined as circular motion. Suppose that an object in the  $xy$ -plane given in the Figure 9.1 rotates around the  $z$ -axis that passes through point  $O$  and is perpendicular to the  $xy$ -plane. The orbits of the  $A$  and  $B$  points on the object will be in the form of rings shown with a dashed line in the figure. The velocity vectors of points  $A$  and  $B$  will always be tangent to these rings. The movements made by the  $A$  and  $B$  points are called **circular motion**. The distances of points  $A$  and  $B$  to point  $O$  are constant throughout the motion. The angle made by the displacement vectors of points  $A$  and  $B$  with the  $x$  or  $y$  axis changes with time. However, depending on whether the velocities vary in magnitude, the motion of the object can be analyzed in two ways. These are Uniform Circular Motion and Nonuniform Circular Motion, respectively.

If the magnitude of the linear velocity of point  $A$  or  $B$  is constant throughout the motion, this motion is called **uniform circular motion**. ( $|\vec{v}_A| = |\vec{v}'_A|$ )

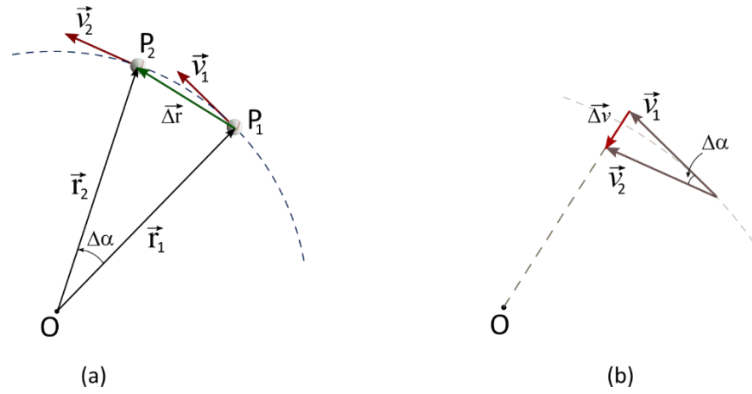
If the direction of the speed of an object moving in a circular motion, as well as the magnitude of its speed, changes over time, this motion is called an **nonuniform circular motion**. ( $|\vec{v}_A| \neq |\vec{v}'_A|$ )



**Figure 9.1** Linear velocities of a circular object at different points.

In uniform circular motion, the direction of the velocity constantly changes, even if the velocities of  $A$  and  $B$  are equal in magnitude throughout the motion. In this case, these points have an acceleration according to Newton's 2nd law ( $\vec{F} = m \frac{d\vec{v}}{dt}$ ).

Let us examine a point particle of mass  $m$  that makes a uniform circular motion in a circular orbit of radius  $R$  around the center  $O$  as shown in the Figure 9.2 below. Suppose the particle moves with a distance  $\Delta s$  from  $P_1$  to  $P_2$  in the time interval  $\Delta t$ . The change in the velocity of the particle during this time will be  $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$ .



**Figure 9.2** Angular displacement of a circular object.

In uniform circular motion, the radius and velocity vectors are  $|\vec{r}_1| = |\vec{r}_2| = r$  and  $|\vec{v}_1| = |\vec{v}_2| = v$ . The triangles shown in the figure 9.2 are similar. Because of the edge-angle-edge similarity, the ratios of the corresponding side lengths are equal. Therefore, equation 9.1 can be written.

$$\frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r} \rightarrow |\Delta \vec{v}| = v \frac{\Delta r}{r} \tag{9.1}$$

Since average acceleration will be the ratio of the change in velocity to the change with time, it can be written as in Equation 9.2.

$$a_{ort} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v \Delta r}{r \Delta t} \tag{9.2}$$

$$\Delta t \rightarrow 0 \text{ ise } \Delta r = \Delta s$$

$$a_{rad} = \lim_{\Delta t \rightarrow 0} \left( \frac{v \Delta s}{r \Delta t} \right) = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta s}{\Delta t} \right) = \frac{v^2}{r} \tag{9.3}$$

As shown in the Figure 9.2, the direction and direction of acceleration  $\vec{a}$  will be the same as  $\Delta \vec{v}$ . The subscript "rad" (defines the radial axis) indicates that the direction of instantaneous acceleration is always in the direction of the radius at each point of the trajectory and is directed towards the center of the circle perpendicular to the velocity and trajectory. Hence it is called **radial acceleration** or **centripetal acceleration**.

According to Newton's 2nd law of motion (eq. 9.4), we can say that a force is applied to the object in the same direction as the acceleration. This force is called the **centripetal force** and its orientation is towards the center of rotation.

$$F = m \frac{v^2}{r} \quad (9.4)$$

The concept of velocity in circular motion is expressed in two forms, **angular velocity** and **linear velocity** (also known as **tangential velocity** due to the possibility of tangent to orbit). Figure 9.1 shows the linear velocities of points A and B at different times on the object making a circular motion.

In circular motion, the velocity can be expressed by the change of the angle of the displacement vector of the object with the x-axis with respect to time. The variation of the swept angle with time is called angular velocity and is denoted by  $\vec{\omega}$ . **Angular Velocity** can be defined as the angular displacement in a certain time interval. Angular velocity, a vector quantity, is in the direction of the rotation axis and its direction is determined by the right-hand rule. Its equivalent in the SI unit system is **rad / s**.

**Right-hand-rule:** Take the axis of rotation in your right palm with your fingertips pointing in the direction of rotation and take a turn in the direction of rotation. The direction of your thumb gives the direction of the angular velocity vector. *The angular velocity vector is taken as positive if the solid object rotates counterclockwise, negative if it rotates clockwise.*

$$\vec{\omega} = \frac{\vec{\theta}_2 - \vec{\theta}_1}{t_2 - t_1} = \frac{\Delta\vec{\theta}}{\Delta t} \quad (9.5)$$

In addition to these, other concepts related to uniform circular motion are:

**Period (T):** It is defined as the time it takes for an object that makes a uniform circular motion to perform a complete rotation and its unit is seconds (s).

**Frequency (f "or v"):** It is the number of cycles per unit time and its unit is  $s^{-1}$  or Hertz (Hz). Frequency and period are inversely proportional ( $f = 1/T$ ).

The linear velocities of the points with different distances to the axis of rotation on an object making a uniform circular motion are different, but the angular velocities of all points on the object are equal. In Equation 9.6 below, the relations between angular velocity, linear velocity, frequency and period are given.

$$w = \frac{2\pi}{T}, \quad w = 2\pi f, \quad v = \frac{2\pi}{T} r = wr \quad (9.6)$$

Centripetal acceleration and centripetal force can also be written in terms of angular velocity as follows.

$$a = \frac{v^2}{r} = w^2 r, \quad F = mw^2 r \quad (9.7)$$



$$a = \left(\frac{2\pi}{T}\right)^2 r, \quad F = m \left(\frac{2\pi}{T}\right)^2 r$$

### 9.4 Tools to be used to in the experiment

Centripetal Force system, Gripper foot, Stopwatch, DC motor and power source, Mass and mass holder, Spool and rope



Figure 9.3 Tolls to be used in experiment.

### 9.5 Experimental Procedur

In this experiment we will examine the centripetal force acting on an object (hooked mass) in uniform circular motion. For this purpose, set up the experimental setup shown in the Figure 9.4 and ensure that all ropes are parallel to the relevant surfaces.

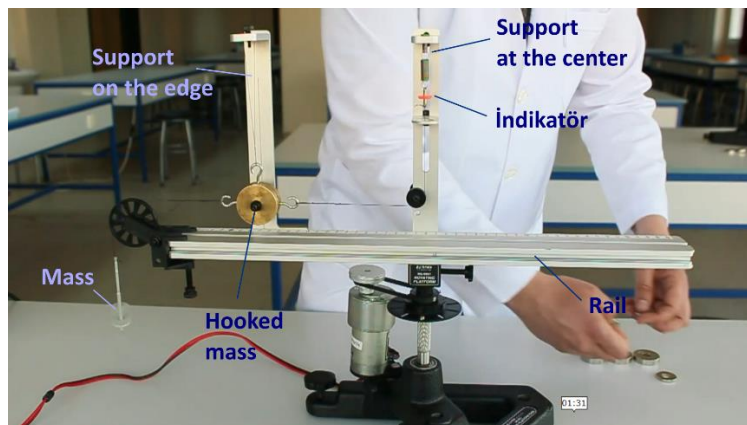


Figure 9.4 Experimental setup

#### 1. Step ( r : constant, m : variable )

1. Determine the location of the indicator (any place you choose arbitrarily on the center support, taking care that the spring does not deform by overstretching) at the end of the spring with the

help of the mass you put in the mass holder that you attach to the free hook of the hooked mass. Weigh the mass you have hung with the mass holder and record it in Table 9.1.

2. Remove the suspended mass with the mass retainer from the hooked mass. In this case, the hooked mass will slide towards the center support.
3. Start the engine and observe the circular motion of the experimental setup. Increase the voltage applied to the motor gradually so that the indicator comes to the place you specified in the previous stage.
4. After waiting for a while, start the stopwatch and measure the time it takes for the setup to rotate 10 rounds and calculate the period and record it in Table 9.1.
5. Then calculate the centripetal force and record it in Table 9.1.

To find the mass of the hooked object

$$mg = M \frac{v^2}{r} = M \frac{4\pi^2 r^2}{T^2 r} \rightarrow M = \frac{mgT^2}{4\pi^2 r} \quad (9.8)$$

Use the expression and calculate the error made taking into account the error calculation.

After determining the period of the rotating object in this way, calculate the angular velocity, linear velocity, frequency and centripetal acceleration values for the object by measuring the radius and write the following equivalents together with their units.

M is the value of the mass with hook and m is the mass value attached to the mass holder (together with the mass holder).

**Table 9.1** Results

$$r = ( \quad \pm \quad ) \quad M_{\text{real}} = ( \quad \pm \quad )$$

Measurement	$m$ (kg)	$T$ (s)	$f$ (1/s)	$\omega$ (rad/s)	$v$ (m/s)
1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
2	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
3	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$

Measurement	$a$ (m/s <sup>2</sup> )	$F_{\text{centripetal}}$ (N)(= $mg$ )	$M$ (kg) (Calculate)
1	$\pm$	$\pm$	$\pm$
2	$\pm$	$\pm$	$\pm$
3	$\pm$	$\pm$	$\pm$

Compare the weight of the mass used to balance the indicator with the centripetal force value you find.

Are your results the same?

### Step 2: (m : constant, r : variable )

After determining the period of the rotating object in this way, calculate the angular velocity, linear velocity, frequency and centripetal acceleration values for the object by measuring the radius and write the following equivalents together with their units.

**Table 9.2 Results**

$$m = ( \quad \pm \quad )$$

Ölçüm	$r$ (m)	$T$ (s)	$f$ (1/s)	$\omega$ (rad/s)	$v$ (m/s)
1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
2	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
3	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$

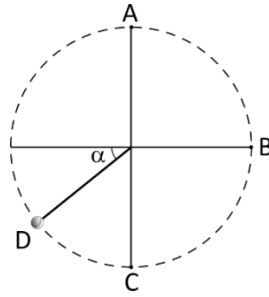
Measurements	$a$ ( $m/s^2$ )	$F_{centripetal}$ (N)(= $mg$ )	$M$ (kg) (Calculate)
1	$\pm$	$\pm$	$\pm$
2	$\pm$	$\pm$	$\pm$
3	$\pm$	$\pm$	$\pm$

Which physical parameter remained constant at this stage of the experiment?

Has the magnitude of the centripetal force changed?

How did the period and velocity change depending on  $r$ ? Discuss.

## 9.6 Post-experiment questions



**Figure 9.5** Tennis ball rotating in a vertical vertical plane.

1. A student spins a tennis ball tied to the end of a ball in a vertical plane. Draw the free body diagram for the ball separately for points A, B, C, and D shown in the figure. What can you say about the velocity of the object at these points?
2. What magnitudes do not change over time in a uniform circular motion?
3. What is the reason for the red indicator to move when the system starts to make circular motion?
4. How can it be said that the centripetal force changes with radius? Has it been observed in this experiment?

## 9.7 References

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# Experiment 9

## Moment of Inertia

DEU Faculty of Science

Physics Department

2021

101

### 10.1 Preparation Questions

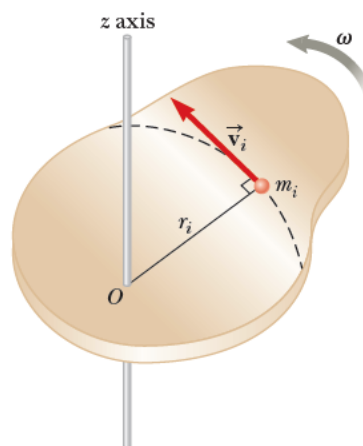
1. Explain the concept of moment of inertia. Discuss whether it is related to Newton's principle of inertia.
2. Define the concept of angular momentum. Explain how its direction is determined.

### 10.2 Aims of the experiment

Investigation of the concepts of moment of inertia, torque and angular momentum in circular motion.

### 10.3 Rotational Kinetic Energy and Moment of Inertia

The measure of the resistance of an object against the rotational motion is called the **moment of inertia**. Suppose a solid body rotating about an axis with an angular velocity  $\omega$  consists of many small particles, each with a mass  $m$ . And let  $r_i$  be the distance of each particle from the axis of rotation.



**Figure 10.1** A rigid body rotating with a constant angular velocity  $\omega$

When a rigid body rotates about a fixed axis, the linear velocity of particle  $i$  is expressed by the equation  $v_i = r_i\omega$ . Different particles have different values of  $r$ , but the  $\omega$  is the same for all (otherwise the body would not be solid). Kinetic energy of particle  $i$ :

$$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2 \quad (10.1)$$

The total kinetic energy of the solid body is the sum of the kinetic energies of all the particles:

$$K = \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + \dots) \omega^2 = \frac{1}{2} \left[ \sum_i m_i r_i^2 \right] \omega^2 \quad (10.2)$$

This relation is valid for discrete systems. The quantity in parentheses, the sum of the mass of each particle multiplied by the squares of their distance from the axis of rotation, is called the "moment of inertia" of the rigid body.

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots = \sum_i m_i r_i^2 \quad (10.3)$$

If the object is assumed to have a continuous structure, the sum symbol in Equation (10.3) is replaced by the integral sign and by integrating over the whole object, the moment of inertia is expressed by

$$I = \int r^2 dm \quad (10.4)$$

Here  $dm$  defines the mass of the infinitesimal part of the object and  $r$  is the perpendicular distance of this part from the axis of rotation. The moment of inertia is a quantity that depends on the shape of the object, its mass distribution, and its axis of rotation. The unit of moment of inertia in the SI system is  $kg \cdot m^2$ .

The total kinetic energy of a body rotating about an axis, given by Equation (10.2), is expressed as:

$$K = \frac{1}{2} I \omega^2 \quad (10.5)$$

As can be seen from equation (10.5), the greater the moment of inertia, the greater the kinetic energy of the rigid body rotating with angular velocity  $\omega$ .

While it is very difficult to calculate the moment of inertia of a body that does not have a definite geometric shape with the help of Equation (10.4), it is quite easy to calculate the moments of inertia of solid bodies with simple shapes. For example, the moment of inertia of a regular disc of radius  $R$  and mass  $M$  about an axis perpendicular to the plane of the disc passing through its center is given by the following equation:

$$I = \frac{1}{2} MR^2 \quad (10.6)$$

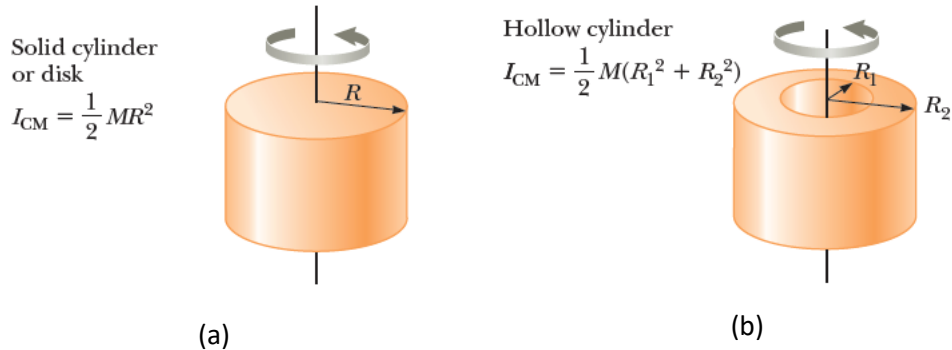
If the axis of rotation of the same disc is chosen as in Figure (10.4), the moment of inertia is expressed by the following equation:

$$I = \frac{1}{4} MR^2 \quad (10.7)$$

A ring of inner radius  $R_1$  and outer radius  $R_2$  has a moment of inertia is defined by the equation



$$I = \frac{1}{2}M(R_1^2 + R_2^2) \quad (10.8)$$



**Figure 10.2** Moments of inertia of (a) Solid cylinder ve (b) hollow cylinder

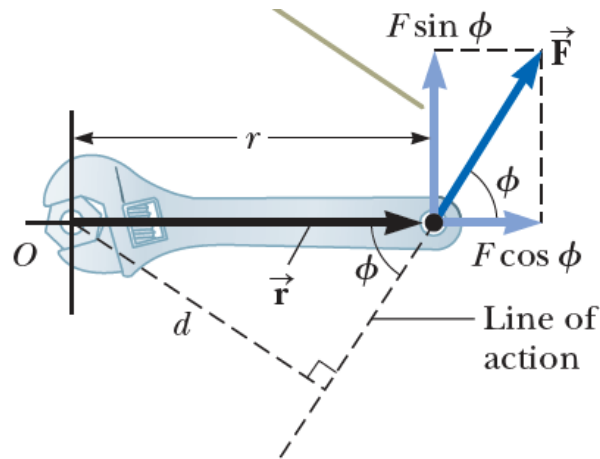
### 10.4 Torque

When a force is applied to an object placed on an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about an axis is measured by a physical quantity called torque ( $\tau$ ).

The force applied to turn a wrench about the O-axis, as in Figure 10.3, can generally make an angle  $\phi$  with the horizontal. The magnitude of the torque resulting from the applied force  $\vec{F}$  is expressed by the following equation:

$$\tau \equiv r F \sin \phi = Fd \quad (10.9)$$

$d = r \sin \phi$ , indicates the perpendicular distance from the axis of rotation to the direction of the force and it is called the moment arm (or force arm) of the  $\vec{F}$  force. The only component of the  $\vec{F}$  force that causes rotation is the component perpendicular to  $r$  is  $F \sin \phi$ . The horizontal component ( $F \cos \phi$ ) passes through point O and has no rotation effect. In order to make an object rotate about an axis, the direction of the force applied to the object and where it is applied are important.



**Figure 10.3** Force applied to the wrench rotated about the O axis

When a force  $\vec{F}$  acts on a point at a distance defined by the position vector  $\vec{r}$  with respect to point O, the torque of that force with respect to point O is a vector quantity:

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (10.10)$$

The unit of torque in the SI system is N.m.

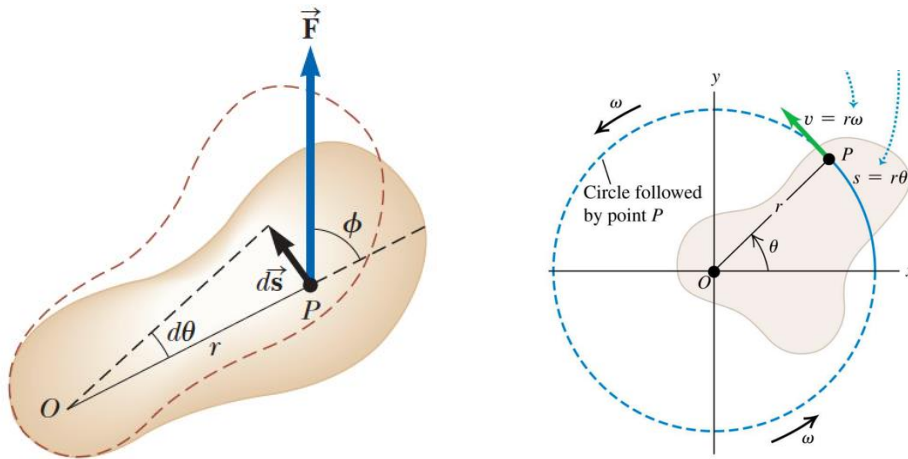
### 10.5 Work and Energy in Rotational Motion

As in Figure 10.4, with the effect of a  $\vec{F}$  force applied on the object from the P point, the object will start to rotate around the O axis and the object will travel on the circular orbit  $ds = r d\theta$  in time dt. In this case, the work done by the force  $\vec{F}$  is:

$$dW = \vec{F} \cdot d\vec{s} = (F \sin \phi) r d\theta \quad (10.11)$$

If we use the torque expression given in Equation (10.9) in Equation (10.11), we get the work done for  $d\theta$  rotation as follows:

$$dW = \tau d\theta \quad (10.12)$$



**Figure 10.4** A rigid body rotates about an axis through  $O$  under the action of a force  $\vec{F}$  applied to point  $P$ .

The kinetic energy of a rotating body was obtained in Equation (10.5) as  $K = \frac{1}{2}I\omega^2$ . Time dependent change of kinetic energy:

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{2}I \frac{d}{dt}\omega^2 = I\omega \frac{d\omega}{dt} \quad (10.13)$$

Since angular acceleration is defined by  $\alpha = d\omega/dt$ ,

$$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\alpha\omega$$

Time-dependent change of work done From Equation (10.10) and definition of angular velocity

( $\omega = d\theta/dt$ ):

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

As a result, the following equation for torque can be written from these last two equations

$$\tau = I\alpha \quad (10.14)$$

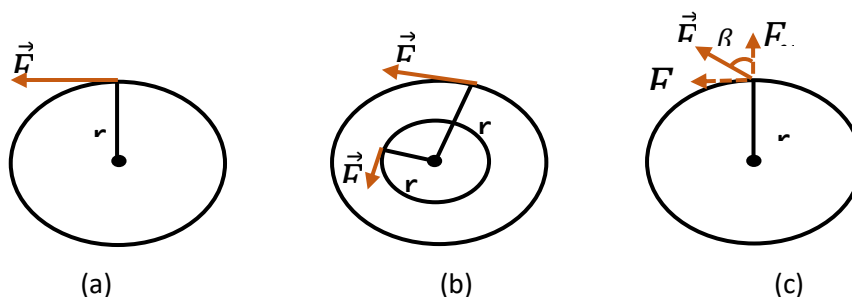
The product of the angular acceleration of an object rotating about a fixed axis and the moment of inertia is equal to the torque acting on the object. The equation (10.14) obtained for rotational motion can be compared to the relation  $F = ma$  in translational motion. Therefore, the mass  $m$  in the translational motion is replaced by the moment of inertia  $I$ , which expresses the mass distribution in the rotational motion.

Figure 10.5 shows a circle passing through the center and rotating around an axis perpendicular to itself (the rotation axis). The force  $\vec{F}$  applied perpendicular to the radius causes the circle to rotate around this secant. But the rotation effect of this force depends on two other things besides its magnitude.

- The distance from the point of applied force to the axis of rotation.
- Angle of force direction with radius direction

With a few simple measurements or observations in daily life, you can come to the following conclusions:

- Two forces of the same magnitude that make the same angle with the radius direction, the one whose point of application is farther from the axis of rotation has the greater rotational effect. In Figure 10.5 (b), the rotation effect of the force applied at the distance  $r_2$  from the rotation axis is greater.
- The two forces of the same magnitude applied at the same distance from the rotation axis, the force that makes a larger angle (between  $0 - \pi/2$ ) with the radius direction has a greater rotational effect. You can see in Figure 10.5(c) that when  $\beta = 0$ , the force will have no rotational effect.



**Figure 10.5** Circular motion and force

Based on the above results, the torque (torque) of the force of magnitude  $|\vec{F}|$  that makes an angle  $\beta$  with the radius direction from the distance  $d$  to the rotation axis as follows

$$\tau = Fd\sin(\beta) \quad (10.15)$$

Torque ( $\tau$ ) is the rotational effect of force, and the concept of "*rotational moment*" is used in circular motion instead of force in linear motion.

## 10.6 Angular Momentum

Similarly, the concept of momentum ( $p = mv$ ) in linear motion corresponds to the concept of angular momentum ( $L$ ) in circular motion. The rotation of an object's linear momentum vector about any point is called angular momentum. If the linear momentum vector  $p$  of the body is  $p$  and the position vector  $r$  connecting this vector to the rotation point ( $r$  and  $p$  are perpendicular to each other), the angular momentum vector of the body is:

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = mr\omega r = I\omega \quad (10.16)$$

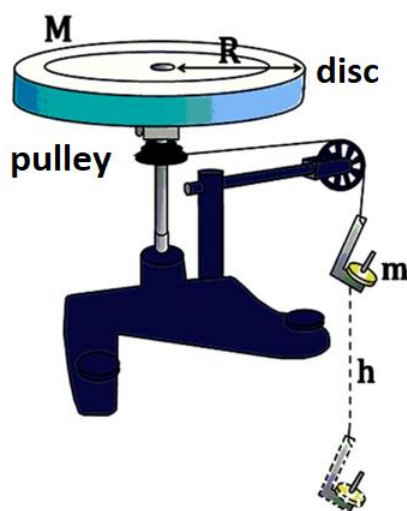
where  $\omega$  is the angular velocity. The SI unit of angular momentum is  $kgm^2/s$ . Angular momentum, similar to linear momentum, is a measure of how difficult it is to change the state of motion of a rotating body.

If no force acts on the rotating object to change its rotational motion, the angular momentum does not change, this is the "conservation of angular momentum". Conservation of angular momentum is one of the important conservation laws of physics and is used in many fields. For example, conservation of angular momentum is often used in analysis in astronomy, atomic and molecular physics, classical and quantum mechanics, nuclear physics, high energy and particle physics.

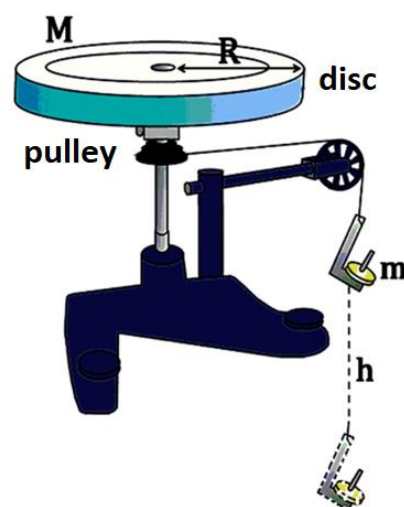
### 10.7 Tools used in experiment

- Circular disc ( $M_{disc}=1500\text{ g}$ )
- Tripod to fix the circular disc
- Rotation axis with discs of different radii on it
- Stopwatch
- Mass holder and different masses
- Ruler, string
- Sliding caliper

### 10.8 Experimental Procedure



(a) Horizontal rotating disc



(b) Vertical rotating disc

**Figure 10.6** Tools used in the experiment

## 10.9 Moment of Inertia

### 10.9.1 Moment of Inertia of Horizontal Rotating Disc

The system shown schematically in Figure 10.6(a) consists of a disc-shaped table that can rotate around a vertical axis and a mass  $m$  falling from height  $h$ . The weight of the mass  $m$  turns the table by pulling the string on a pulley of radius  $r_1$ . The system moves with the effect of a constant force (the effect of the tensile force formed by the mass  $m$ ). Hence, the motion of mass  $m$  is uniformly accelerated linear motion. If the system starts to move from rest,  $v_0 = 0$  and if we consider that the object with mass  $m$  takes the  $h$  path in the vertical plane in  $t_1$  time, we get the linear acceleration of the object as follows:

$$y - y_0 = v_0 t + \frac{1}{2} a t^2 \quad , \quad y - y_0 = h \quad \text{ve} \quad v_0 = 0$$

$$h = \frac{1}{2} a t^2 \quad \Rightarrow \quad a = \frac{2h}{t^2} \quad (10.17)$$

The tension in the string is according to Newton's II. law of motion is

$$T_1 = m(g - a_1) = m\left(g - \frac{2h}{t_1^2}\right) \quad (10.18)$$

In this case, according to the equations (10.10) and (10.14), the torque acting on the rotating disc is:

$$\tau_1 = \vec{d} \times \vec{F} = T_1 r_1 \quad (10.19)$$

$$\tau_1 = I_1 \alpha_1 \quad (10.20)$$

Here  $r_1$  is the radius of the pulley which the string is wrapped and  $\alpha_1$  is the angular acceleration of the system. Using the relationship between the angular and linear accelerations, we can obtain the moment of inertia of the spinning disc from equations (10.17) and (10.20):

$$\alpha_1 = \frac{a_1}{r_1} = \frac{2h}{r_1 t_1^2} \quad (10.21)$$

$$I_1 \alpha_1 = T_1 r_1 \Rightarrow I_1 = m\left(g - \frac{2h}{t_1^2}\right) \frac{r_1^2}{a_1} \quad (10.22)$$

- Set up the experimental setup as in Figure 10.6 (a) and measure the radius of pulley which the string is wrapped with a slide caliper. Record the radius value.

- Suspend a small mass  $m$  on the mass holder. Weigh the total mass (mass holder and suspended masses) on a precision balance.
- Hold the circular disc with your hand and set the height  $h_1$  (distance from the ground to the mass holder) at which mass  $m$  will be released and measure with a ruler.
- Prepare the stopwatch. Release the disc and start the stopwatch. Measure the descending time of the mass from height  $h_1$  and record it in Table 10.1.
- Measure the mass from the ground in the final state  $h_2$
- Calculate the distance taken by the mass in the vertical plane ( $h = h_1 - h_2$ ).
- Measure the descending time of the mass from the same height  $h_1$  5 times and record it in Table 10.1.
- Calculate the linear acceleration and angular acceleration from equations (10.17) and (10.21).
- Calculate the tension force in the string from equation (10.18).
- Calculate the moment of inertia of the object experimentally using equation (10.22) ( $I_{experimental}$ ).
- Calculate the moment of inertia of the object theoretically using equation (10.6) ( $I_{theoric}$ ).
- Compare the theoretical value you obtained with the experimental value and calculate the error.

### 10.9.2 Moment of Inertia of Vertical Rotating Disc

- Set up the experimental setup as in Figure 10.5 (b) in order to understand how the moment of inertia changes with the change of rotation axis.
- Make the system rotate in a similar way without changing the radius  $r$  around which the string is wrapped, the mass  $m$  suspended and the height at which it is released.
- Measure the descending time of the mass from height  $h_1$  5 times and record it in Table 10.2.
- In this case, the radius of the pulley with string is not changed, the magnitude of the torque will also remain constant:

$$\tau_1 = \tau_2$$
$$I_1\alpha_1 = I_2\alpha_2$$

Calculate the angular acceleration from equation (10.21). Using the results of  $I_1$  and  $\alpha_1$  obtained in the first stage of the experiment, experimentally calculate the moment of inertia of the vertical rotating disc using the equation below ( $I_{experimental}$ ).

$$I_2 = I_1 \frac{\alpha_1}{\alpha_2}$$

- Calculate the theoretical value of the moment of inertia of the object using equation (10.6) ( $I_{theoric}$ ).
- Compare the theoretical value you obtained with the experimental value and calculate the error.
  - Did the disk spin slower or faster in the second stage of the experiment than in the first stage? Why?
  - Are the results found different even though the same disc was used in both stages of the experiment? Explain why.

### 10.9.3 Conservation of Energy

In this part of the experiment, it is aimed to find the moment of inertia with the help of conservation of energy. For this, the energy before the system starts to move and the energy after the system starts should be equal to each other. Since the disk and mass  $m$  are at rest before mass  $m$  is released, the energy in the first case is only the potential energy of mass  $m$ .

$$E_{initial} = mgh \quad (10.23)$$

After the mass  $m$  is released, when it starts to move vertically, the disc will also start to rotate. Therefore, the energy of the disc is the kinetic rotational energy given in Equation (10.5) ( $K = \frac{1}{2}I\omega^2$ ).

Since the mass  $m$  will move simultaneously, the mass  $m$  will also gain kinetic energy. If the final position reference point of the mass  $m$  is selected, the potential energy will be zero, so the following equation can be written for the final state energy:

$$E_{final} = E_{final (mass)} + E_{final (disc)} \quad (10.24)$$

$$E_{final} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad (10.25)$$

By equating the equations (10.23) and (10.25) to each other:

- Calculate the moment of inertia for the horizontal rotating disc.



- Calculate the moment of inertia for the vertical rotating disc.
- Compare the experimental results you have obtained with the theoretical results and calculate the error.

### 10.10 Measurements and Results

First stage of the experiment:

$$r_1 = \dots\dots\dots \text{ m} \qquad m = \dots\dots\dots \text{ kg} \qquad M_{disc} = 1.5 \text{ kg}$$

$$h_1 = \dots\dots\dots \text{ m} \qquad h_2 = \dots\dots\dots \text{ m} \qquad h = h_1 - h_2 = \dots\dots\dots \text{ m}$$

**Table 10.1** Measurement results for horizontal rotating disc

Measurement No.	t (s)	a(m/s <sup>2</sup> )	$\alpha$ (rad/s <sup>2</sup> )	$T_1$ (N)	$I_{experimental}$ (kg.m <sup>2</sup> )
1					
2					
3					
4					
5					
Average Value					
Error					

$$I_{experimental} = (\dots\dots\dots \pm \dots\dots\dots) \text{ kg.m}^2$$

$$I_{theoric} = (\dots\dots\dots) \text{ kg.m}^2$$

Second stage of the experiment:

$$r_1 = \dots\dots\dots \text{ m} \qquad m = \dots\dots\dots \text{ kg} \qquad M_{disc} = 1.5 \text{ kg}$$

$$h_1 = \dots\dots\dots \text{ m} \qquad h_2 = \dots\dots\dots \text{ m} \qquad h = h_1 - h_2 = \dots\dots\dots \text{ m}$$

**Table 10.2** Measurement results for vertical rotating disc

Measurement No.	t (s)	a(m/s <sup>2</sup> )	$\alpha$ (rad/s <sup>2</sup> )	$T_1$ (N)	$I_{experimental}$ (kg.m <sup>2</sup> )
1					
2					
3					
4					
5					
Average Value					
Error					

$$I_{experimental} = ( \dots \pm \dots ) kg.m^2$$

$$I_{theoric} = ( \dots ) kg.m^2$$

Third stage of the experiment:

$$E_{initial} = mgh$$

$$E_{initial} = ( \dots ) J$$

**Table 10.3** Calculations for conservation of energy

Measurement No.	t (s)	v(m/s)	$\omega$ (rad/s)	$E_{final}$ (mass) (J)	$E_{final} (disc) = E_{initial} - E_{final} (mass)$ (J)	$I_{experimental}$ (kg.m <sup>2</sup> )
1						
2						
3						
4						
5						
Average Value						

Error						
-------	--	--	--	--	--	--

$$I_{\text{experimental}} = (\dots \pm \dots) \text{ kg.m}^2$$

$$I_{\text{theoric}} = (\dots) \text{ kg.m}^2$$

### 10.11 Questions

1. What physical quantity corresponds to the moment of inertia in translational motion, explain?
2. If this experiment were done on the moon, would the moment of inertia change? Explain why?
3. Let the force  $\vec{F}$  of equal magnitude in Figure 3(b) applied from the center distance  $r_1$  make an angle  $\beta_1$  with the radius direction, and the one applied from a distance of  $r_2$  make an angle of  $\beta_2$  with the radius direction. Find the relation between  $r_1, r_2, \beta_1, \beta_2$  if both torques are equal.
4. If  $r_1=r_2$  and  $\beta_2 = 90^\circ$ , can the torque of the two forces be equal? Explain why.

